Bayesian Statistics as an Alternative for Analyzing Data and Testing Hypotheses

Benjamin Scheibehenne

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Can Social Norms Increase Towel Reuse?

Standard Environmental Message

Descriptive Social Norm

Reuse:	35.1%	44.1%

$$X^2(1, N = 433) = 3.72; p = .05$$

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Goldstein, Cialdini, & Griskevicius, 2008, *Journal of Consumer Research*

The Null Ritual

1. Set up a statistical null hypothesis of “no mean difference” or “zero correlation.” Do not specify any substantive alternative hypotheses.
2. Calculate a p-value.
3. If p<.05, refer to the result as “statistically significant” and interpret it as if you obtained evidence for the alternative hypothesis.

Gigerenzer, Krauss, & Vitouch (2004)

What is Wrong with this Approach?

- p-values do not quantify evidence for your null or alternative hypothesis
- With high enough sample size, you are guaranteed to get $p<.05$
- p-values depend on the intention of the researcher (identical data can yield different p values)
- p-values are not consistent: Little can be learned from non-significant results

When H_0 is True, p-Values are not Consistent

Alternative hypothesis is true

More data → higher chance of identifying the true hypothesis

Null-Hypothesis is true

More data → chance of identifying the true (null) hypothesis is constant (at 1-α)

Under H_0, p-Values are uniformly distributed

p-Values Depend on the Intention of the Experimenter

- Default assumption: *N* is fixed

$$X^2(1, N = 433) = 3.72;\ p = .05$$

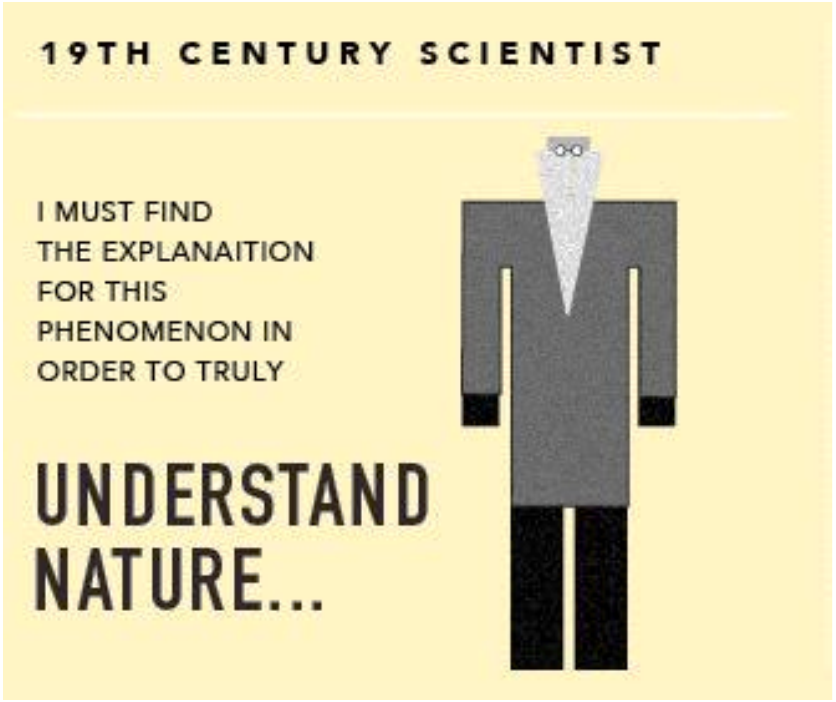
actually: $p = .053$

- What if the number of hotel guests is a random variable?

$N_{control} \sim \mu = 211 \quad \sigma = 30$

$N_{ex} \sim \mu = 222 \quad \sigma = 30$

$$X^2(1, N = 433) = 3.72;\ p = .06$$


19TH CENTURY SCIENTIST
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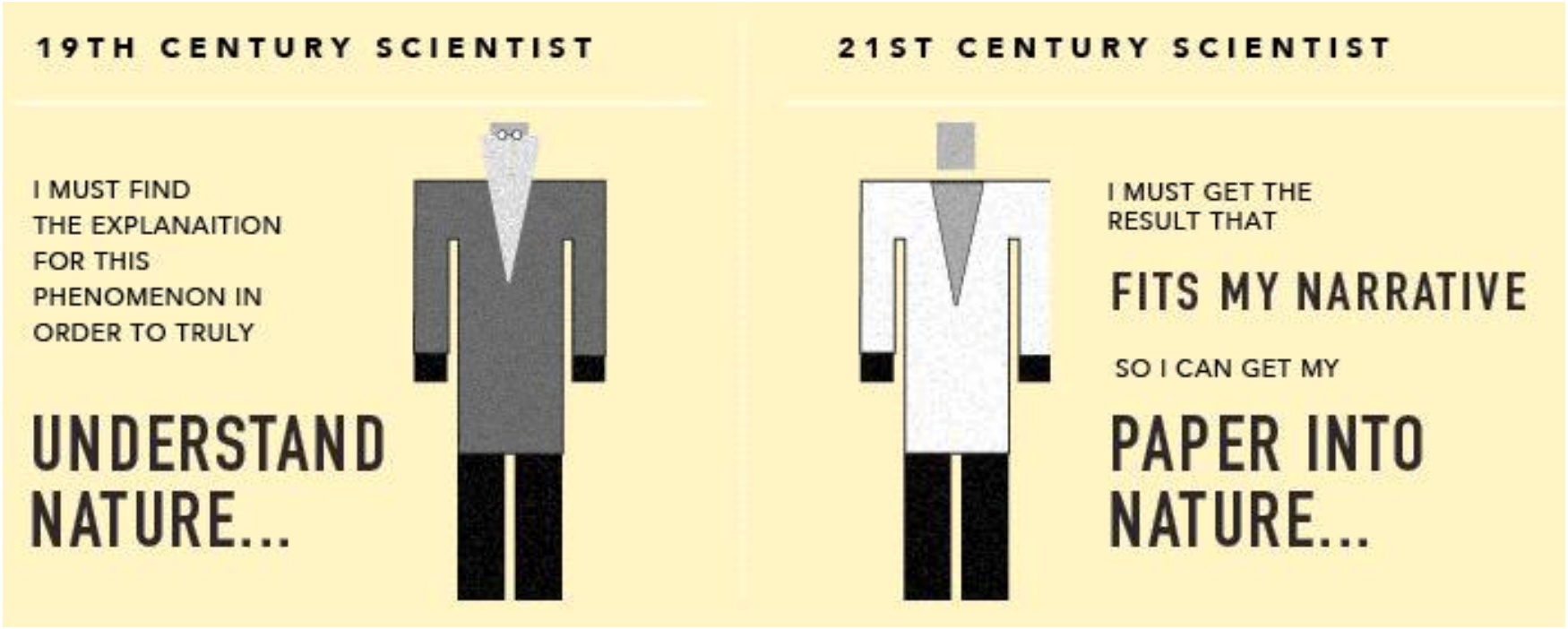
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NATURE...

American Statistical Association Issues Warning

nature International weekly journal of science

Statisticians issue warning over misuse of *P* values

Policy statement aims to halt missteps in the quest for certainty.

"The widespread use of "statistical significance" (generally interpreted as "p ≤ 0.05") as a license for making a claim of a scientific finding (or implied truth) leads to considerable distortion of the scientific process."

Misuse of the *P* value — a common test for judging the strength of scientific evidence — is contributing to the number of research findings that cannot be reproduced, the American Statistical Association (ASA) warns in a statement released today[1]. The group has taken the unusual step of issuing principles to guide use of the *P* value, which it says cannot determine whether a hypothesis is true or whether results are important.

This is the first time that the 177-year-old ASA has made explicit recommendations on such a foundational matter in statistics, says executive director Ron Wasserstein. The society's members had become increasingly concerned that the *P* value was being misapplied in ways that cast doubt on statistics generally, he adds.

Message is not new…

"To report a significant result and reject the null in favor of an alternative hypothesis is meaningless […]." (*Richard Feynman*)

"In the land of the blind, keep your eyes closed."

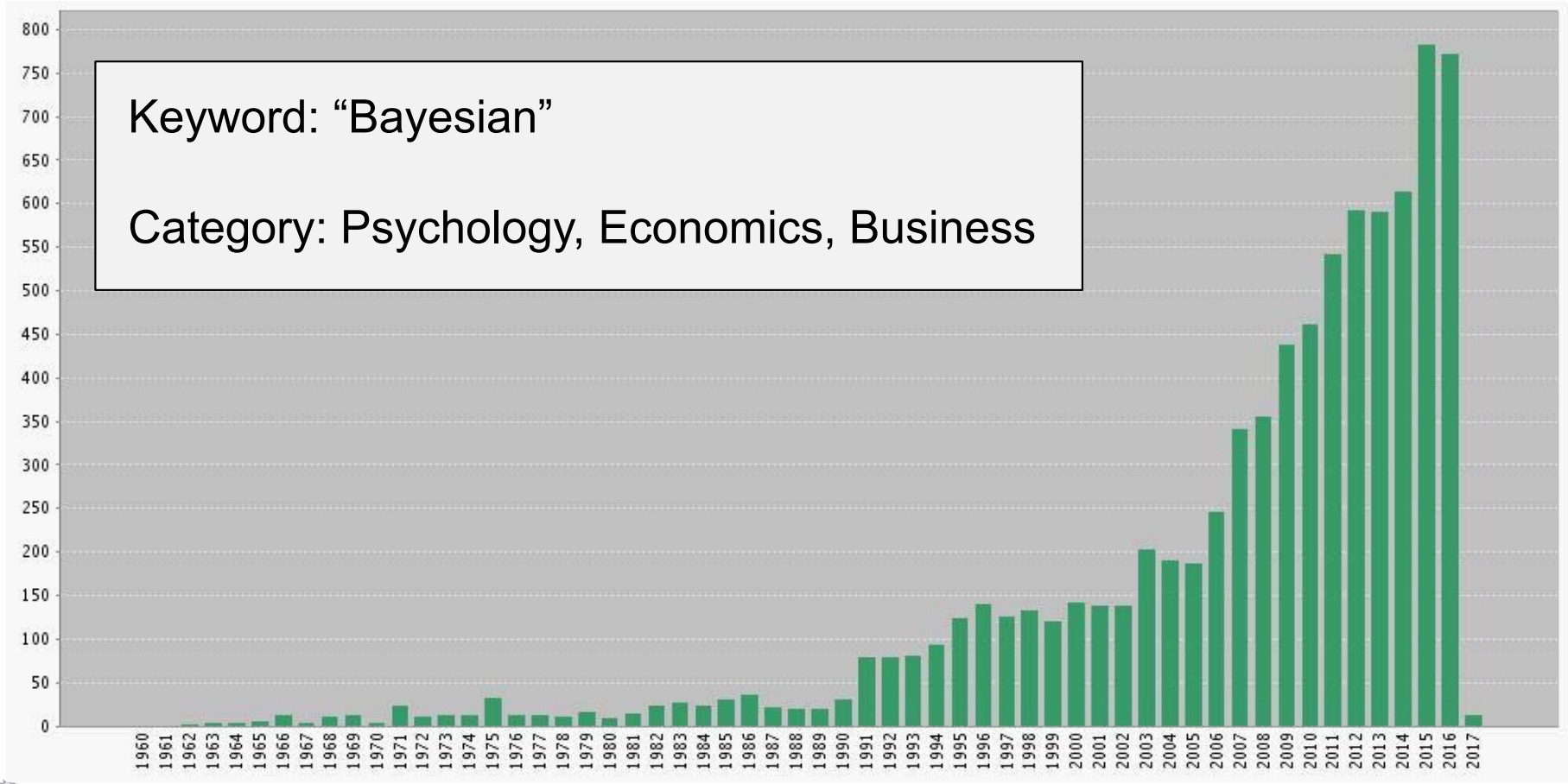
[proverb from Kurdistan]

Bayesian Approach Provides an Alternative

"Friends don't let friends compute p-values"

Bayesian Approach Increasingly Attractive

 # Articles in Web of Science



What Makes the Bayesian Approach Attractive?

New Results
- Evidence for Hypotheses (incl. H0)

Better Results
- More accurate estimates and predictions
- Intuitive interpretation

Compare different models and hypotheses
- Take model complexity into account

Broad application
- “One-stop shop“

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Standard Message

Descriptive Social Norm

What is the evidence?

Ex1:	35.1%	44.1%	N = 433	p = .05
Ex2:	37.2%	44.5%	N = 1,595	p = .03

Bayesian Hypothesis Test

- Suppose we have two hypotheses: H1 and H0
- Which hypothesis is better supported by the data?
- The model that makes the best prediction for the data!
- Ratio of predictive performance = **Bayes Factor**

Bayes Factor

Prior odds

$$\frac{p(H_1)}{p(H_0)}$$

•

Bayes Factor

X

Posterior odds

$$= \frac{p(H_1 \mid D)}{p(H_0 \mid D)}$$

Change in probability of one hypothesis/ model over another after seeing the data

posterior probability of one model over the other

Bayes Factor

Prior odds

$$\frac{p(H_1)}{p(H_0)}$$

$\bullet$

Bayes Factor

$$\frac{\int p(D \mid H_1) \bullet p(H_1)}{\int p(D \mid H_0) \bullet p(H_0)}$$

$=$

Posterior odds

$$\frac{p(H_1 \mid D)}{p(H_0 \mid D)}$$

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Prior odds | **Bayes Factor** | **Posterior odds**

$$\frac{p(H_1)}{p(H_0)} \bullet \frac{\int p(D \mid H_1) \bullet p(H_1)}{\int p(D \mid H_0) \bullet p(H_0)} = \frac{p(H_1 \mid D)}{p(H_0 \mid D)}$$

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posterior probability of one model over the other

Simple Interpretation:

BF = 3: After seeing the data, H_1 became 3 times more probable than H_0

Bayes Factor

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1 to 3	Anecdotal
3 to 10	Substantial
10 to 100	Strong
>100	Decisive

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Advantages of Bayes Factors

- Quantifies evidence instead of forcing an all-or-none decision
- Easy to interpret
- All assumptions are explicit
- Able to distinguish between “data support H_0” and “data are not diagnostic”

Bayes Factor for Towel Data

Ex1	Reuse	Throw
Standard message	74	137
Social norm	98	124

Gunel & Dickey (1974) *Biometrika*

Bayes Factor for Towel Data

Ex1	Reuse	Throw
Standard message	74	137
Social norm	98	124

Assumptions:

- Sample size in each condition (row) is fixed
- Probability of reuse = $\pi_{standard}$, $\pi_{social\ norm}$
- Prior: Each value for π is equally probable a-priori
- H0: $\pi_{standard} = \pi_{social\ norm}$
- H1: $\pi_{standard} < \pi_{social\ norm}$

Independent multinomial distribution

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Independent multinomial distribution

Result:

- BF = 1.46
- BF for undirected hypothesis ($\pi_{standard} \neq \pi_{social\ norm}$) = 0.75

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Ex1:	35.1%	44.1%	N = 433	p = .05	**BF = 1.46**
Ex2:	37.2%	44.5%	N = 1,595	p = .03	**BF = 2.03**

Can Social Norms Increase Towel Reuse?

Lots of failed replications…

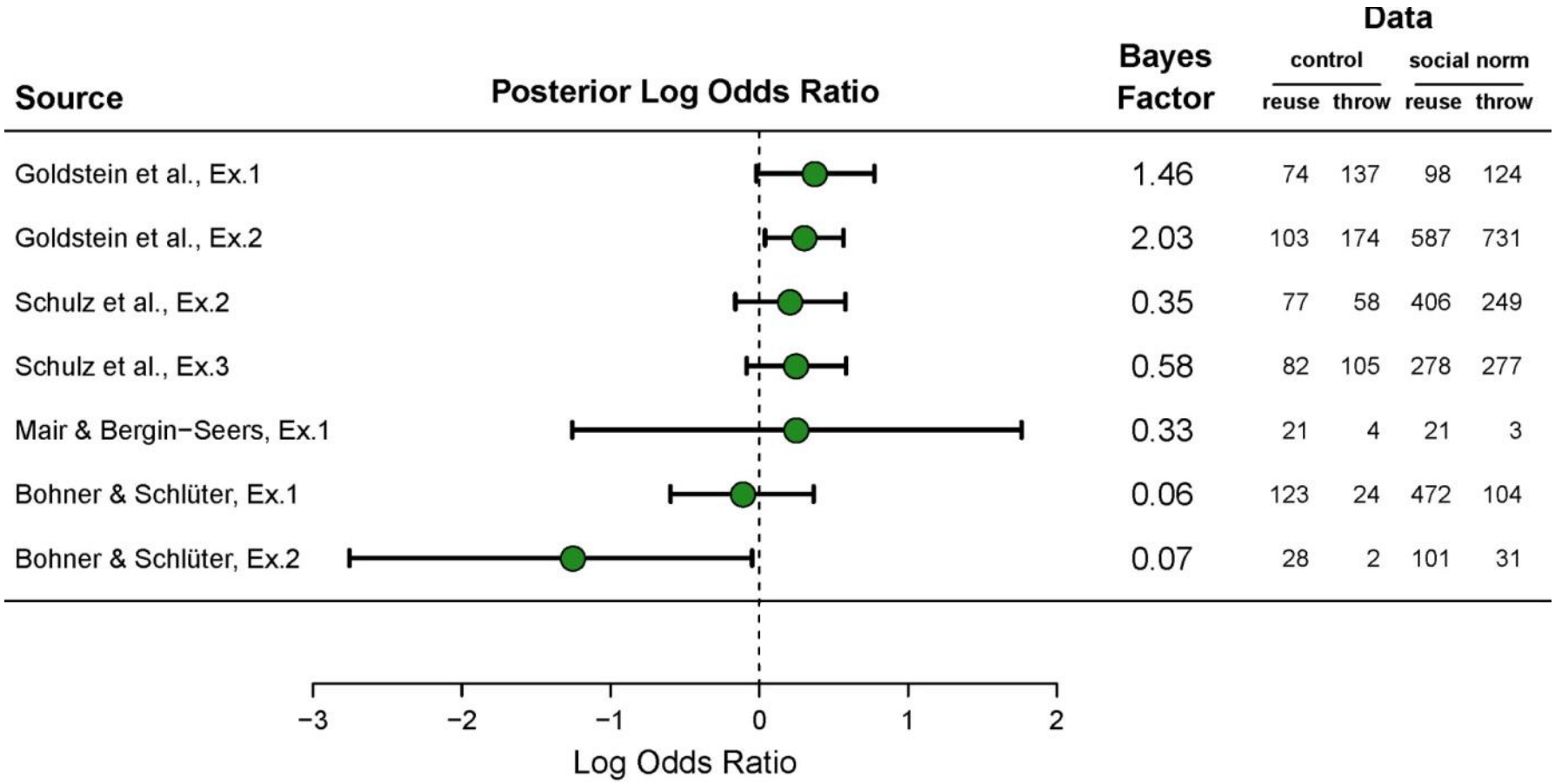
Schultz et al. (2007) *Social Influence*

Mair & Bergin-Seers (2010) *Tourism and Hospitality Research*

Bohner & Schlüter (2014) *PloS ONE*

} all p's > .14

Can Social Norms Increase Towel Reuse?



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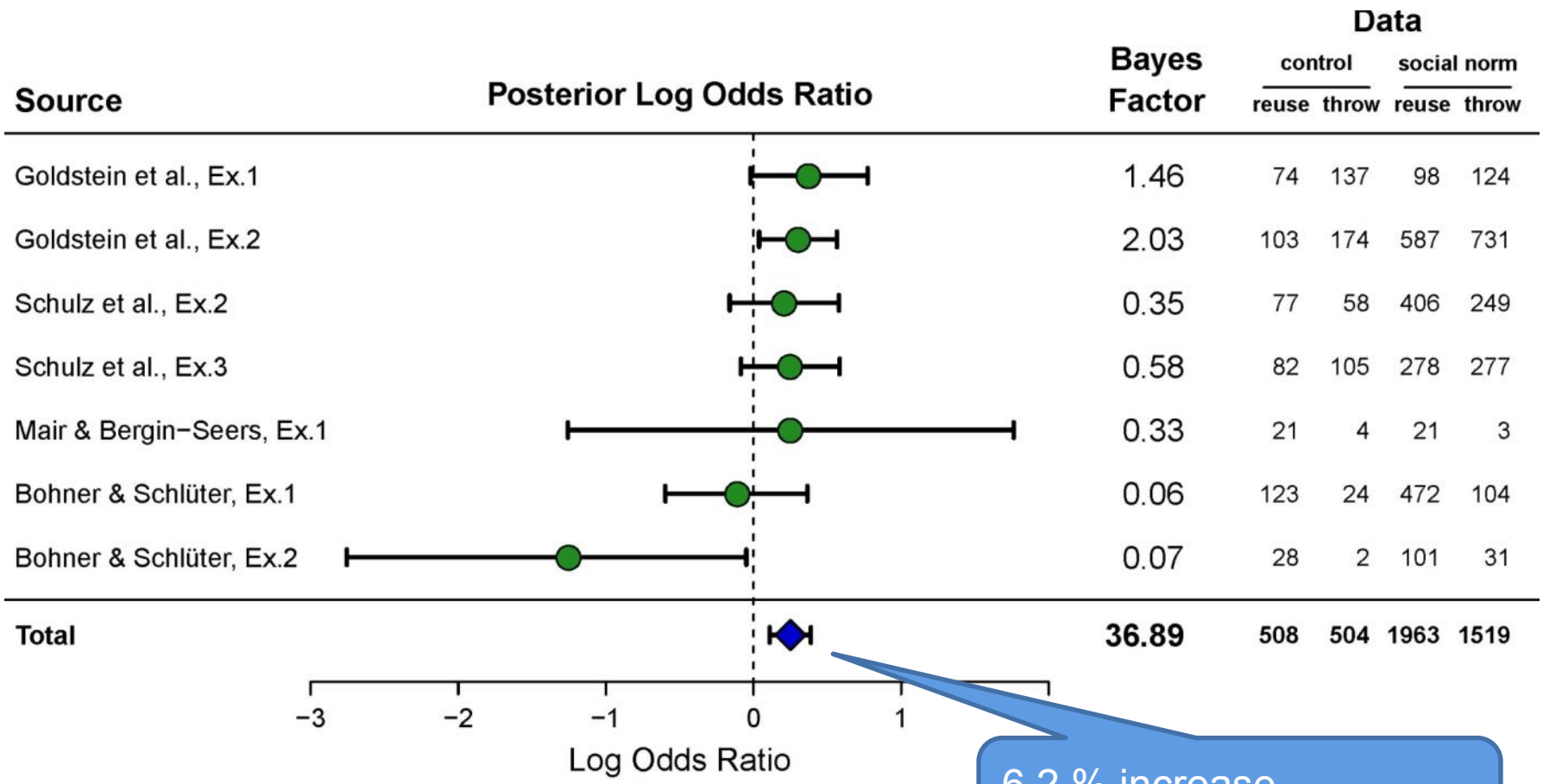

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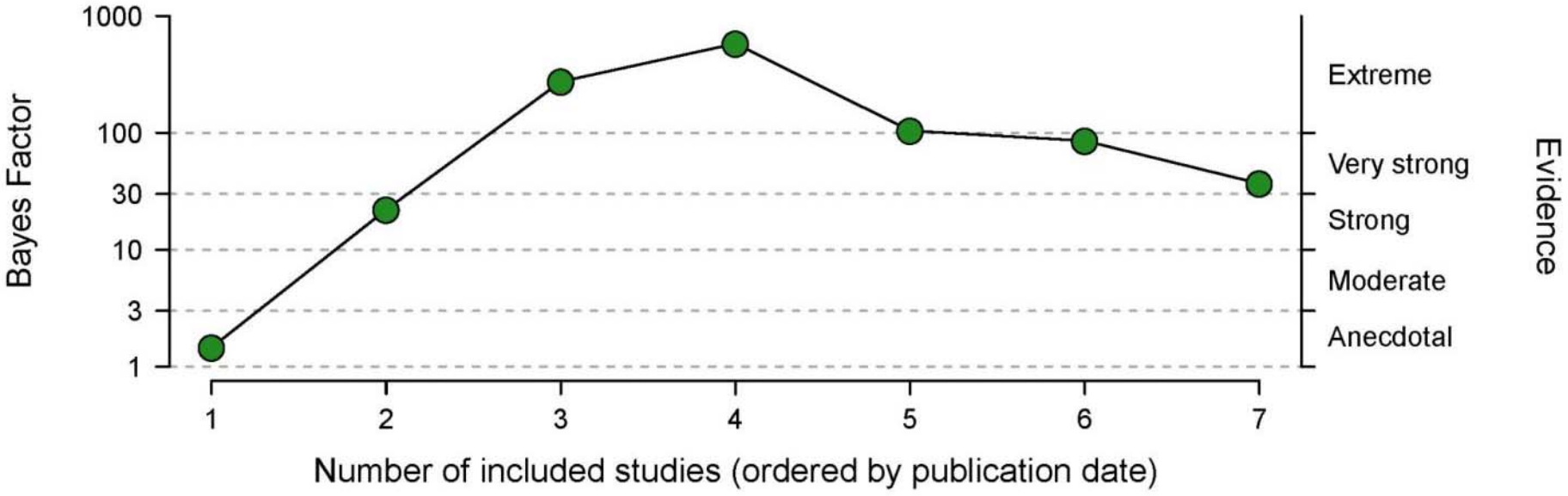
Can Social Norms Increase Towel Reuse?

- Cumulative evidence “as the data comes in”
- Can be updated in the future



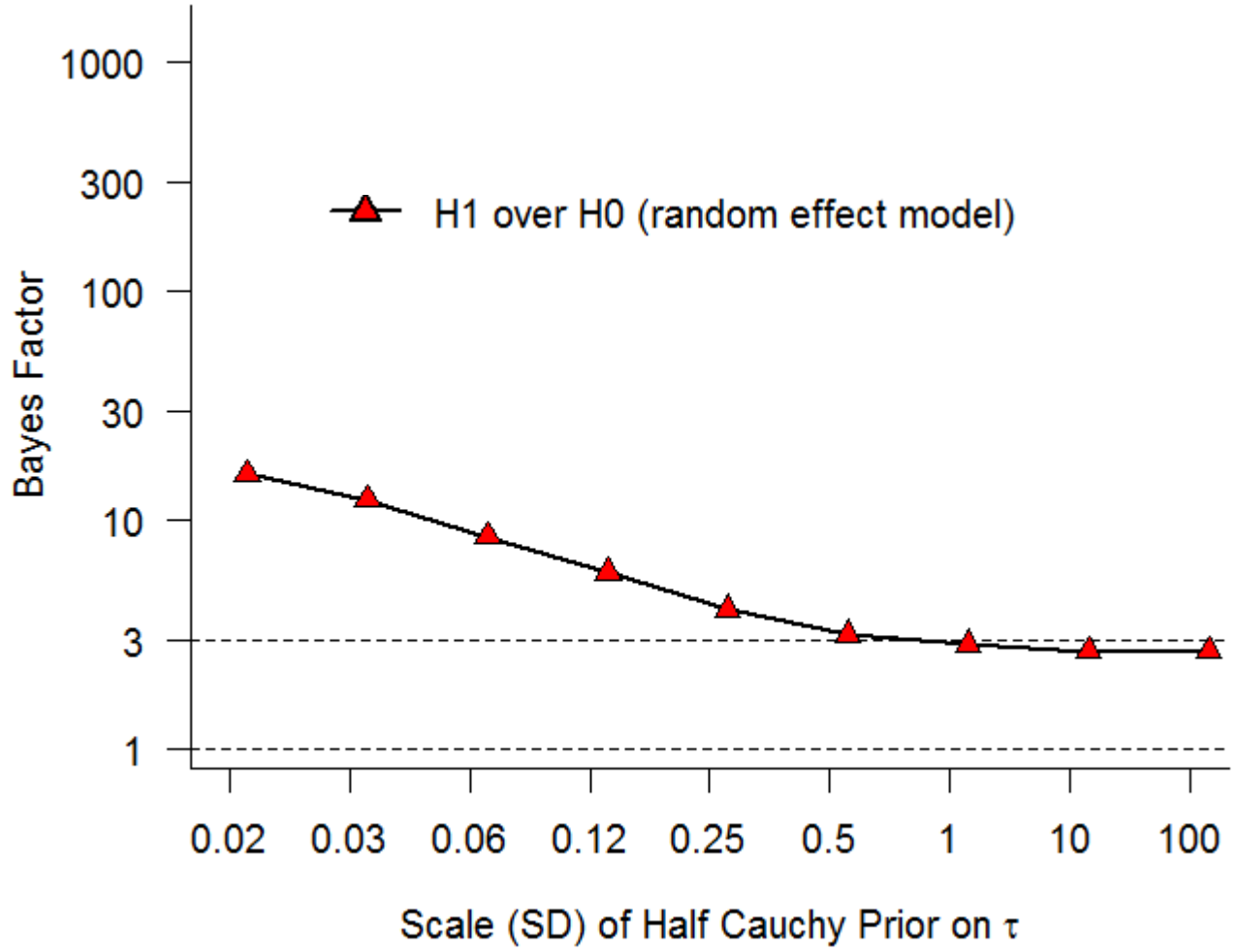
Repeated Testing is Allowed

- Multiple tests in NHST = risk of false-positive findings
- In a Bayesian framework, optional stopping does not distort the results
- No correction for repeated testing required

"It follows that any significance test based on Bayes's theorem does not depend on the sequential stopping rule used […]"

Dennis Lindley (1957)

Would a Random-Effect Model be Better Suited?



= Prior on the between-study variance

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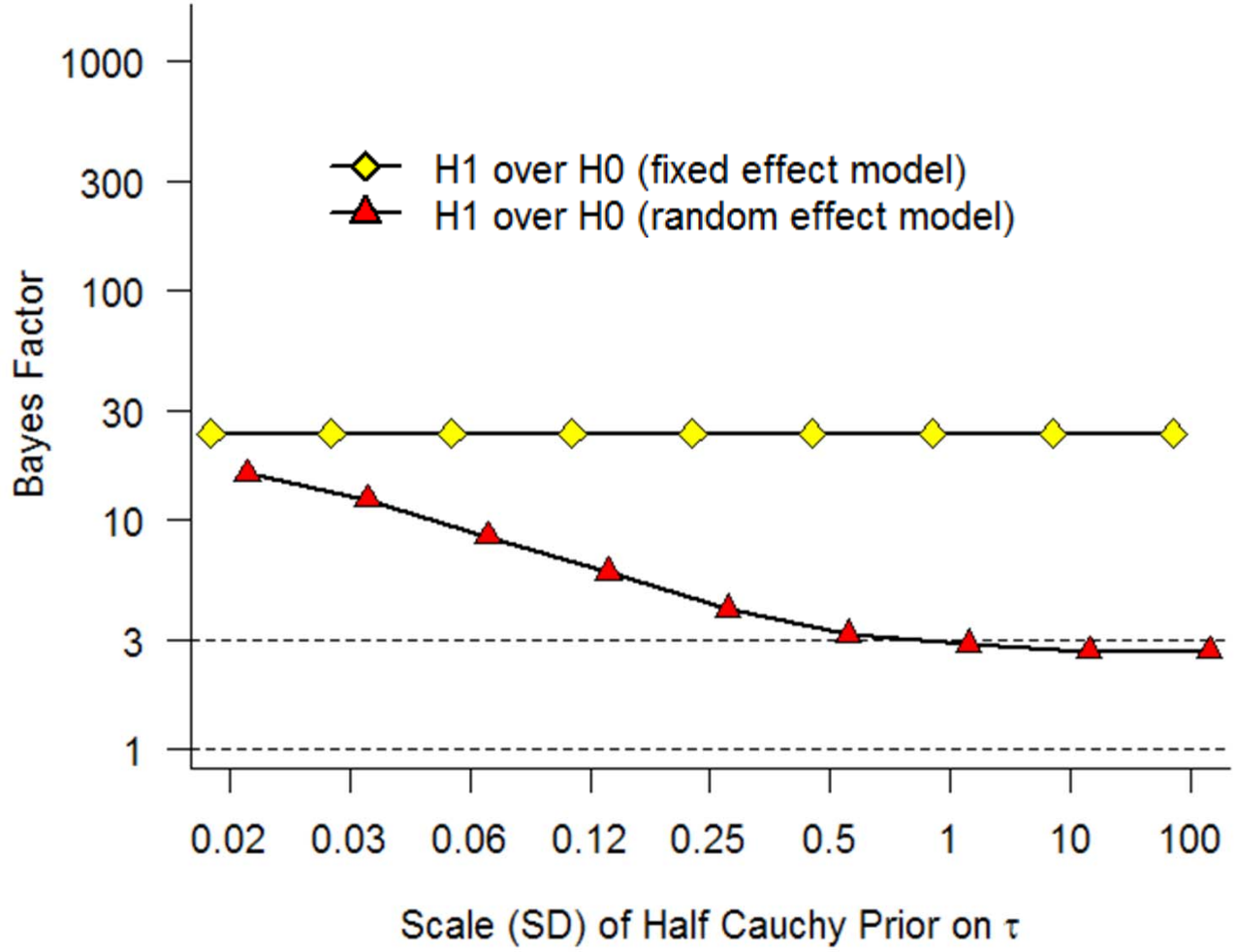

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Broad application
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How can we Compare Models?

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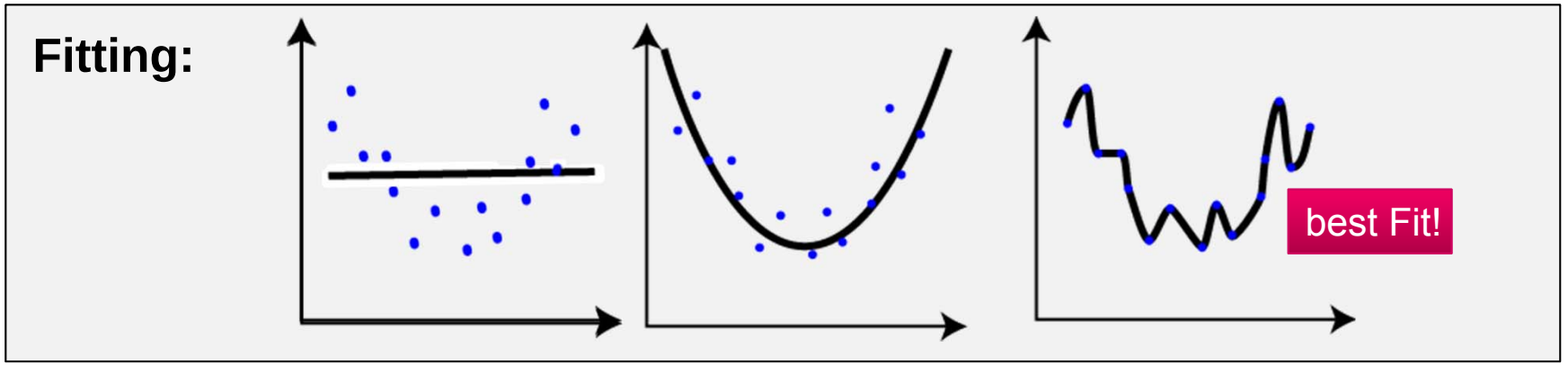
More complex models provide a better *description* of the data...



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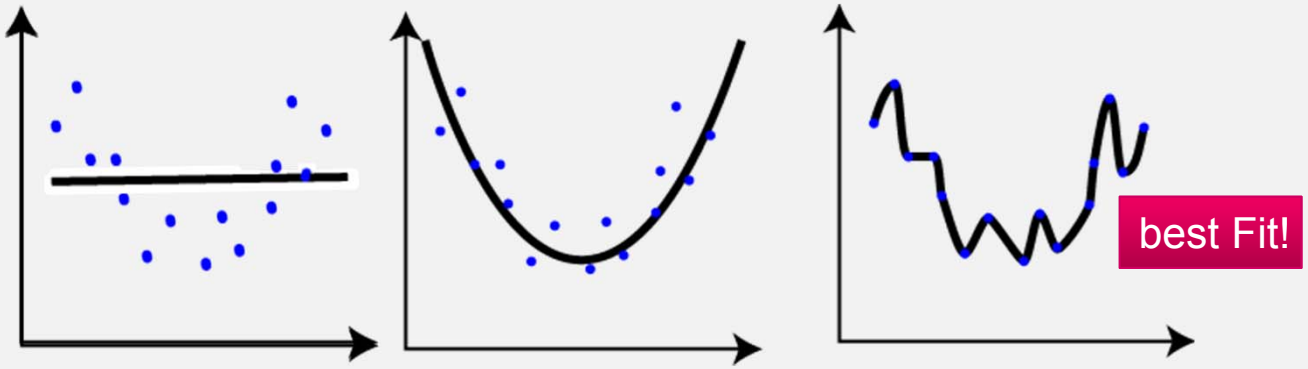
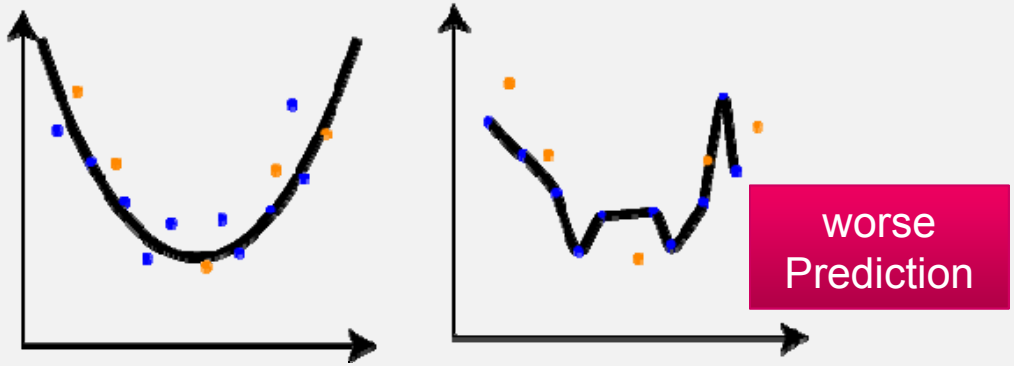
Fitting:



…but not necessarily a better *explanation*:

Prediction of new data:



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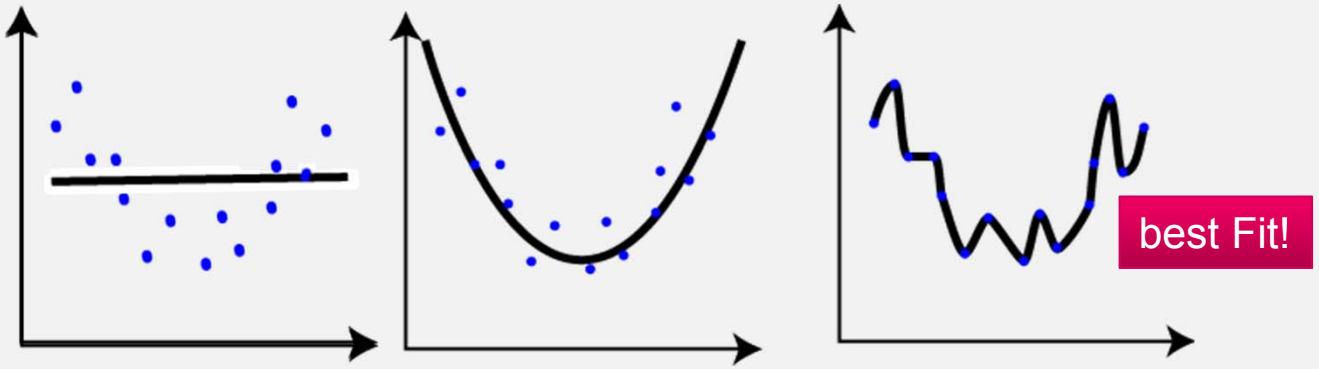

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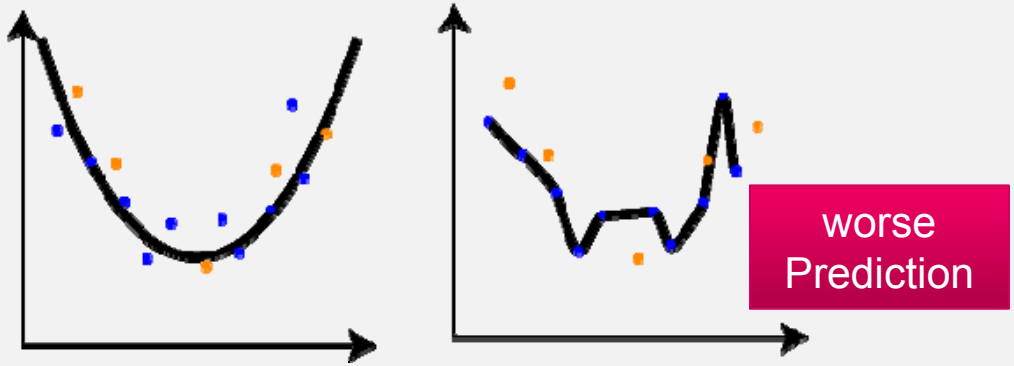

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Trade-off between complexity and fit

Bayes Factor

average probability of the observed data across all predictions the model makes

$$\frac{\int p(D|\theta_1, M_1) \cdot p(\theta_1, M_1)}{\int p(D|\theta_2, M_2) \cdot p(\theta_2, M_2)}$$

- Flexible models make many predictions
- If most of these predictions are wrong, this drives down the average probability

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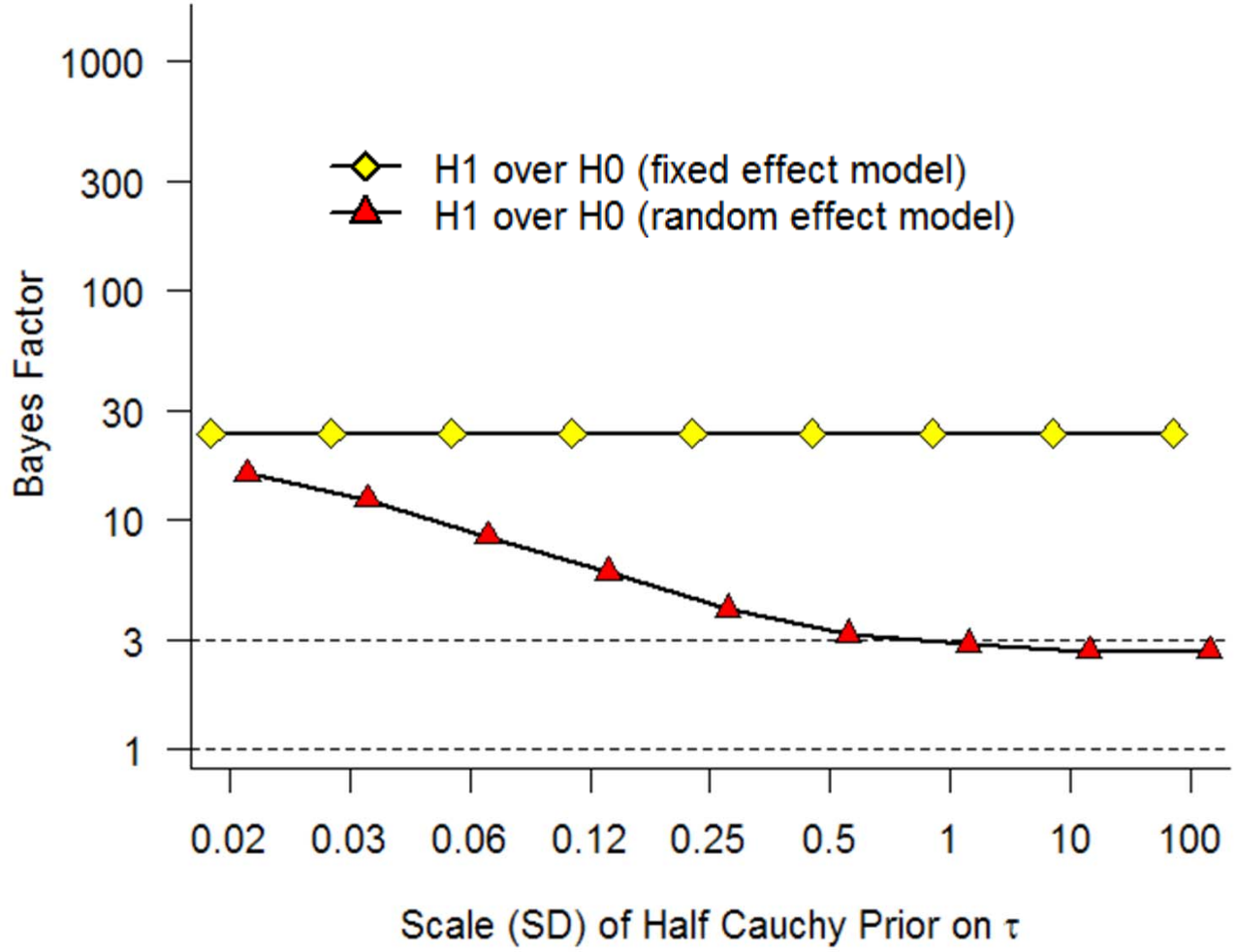

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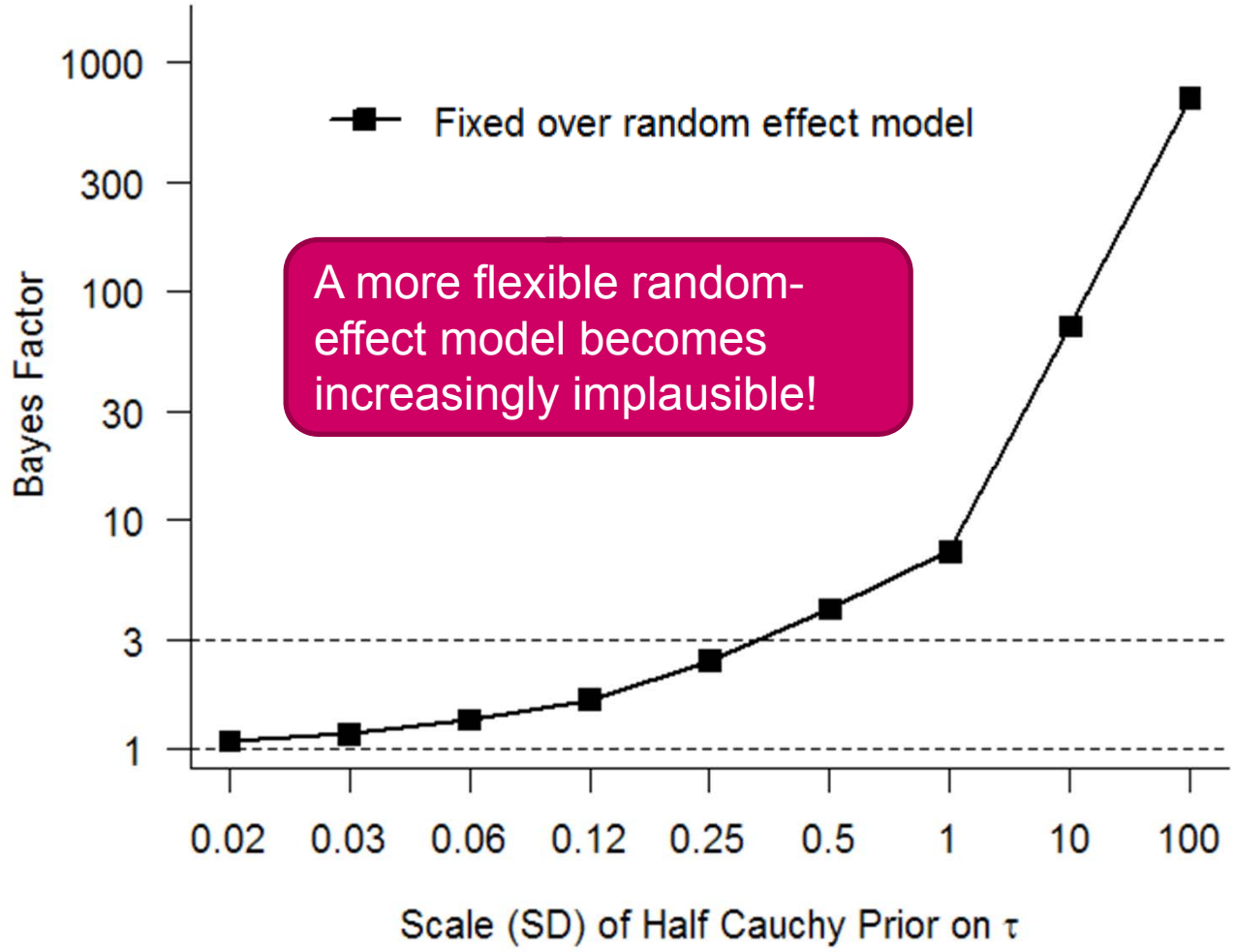

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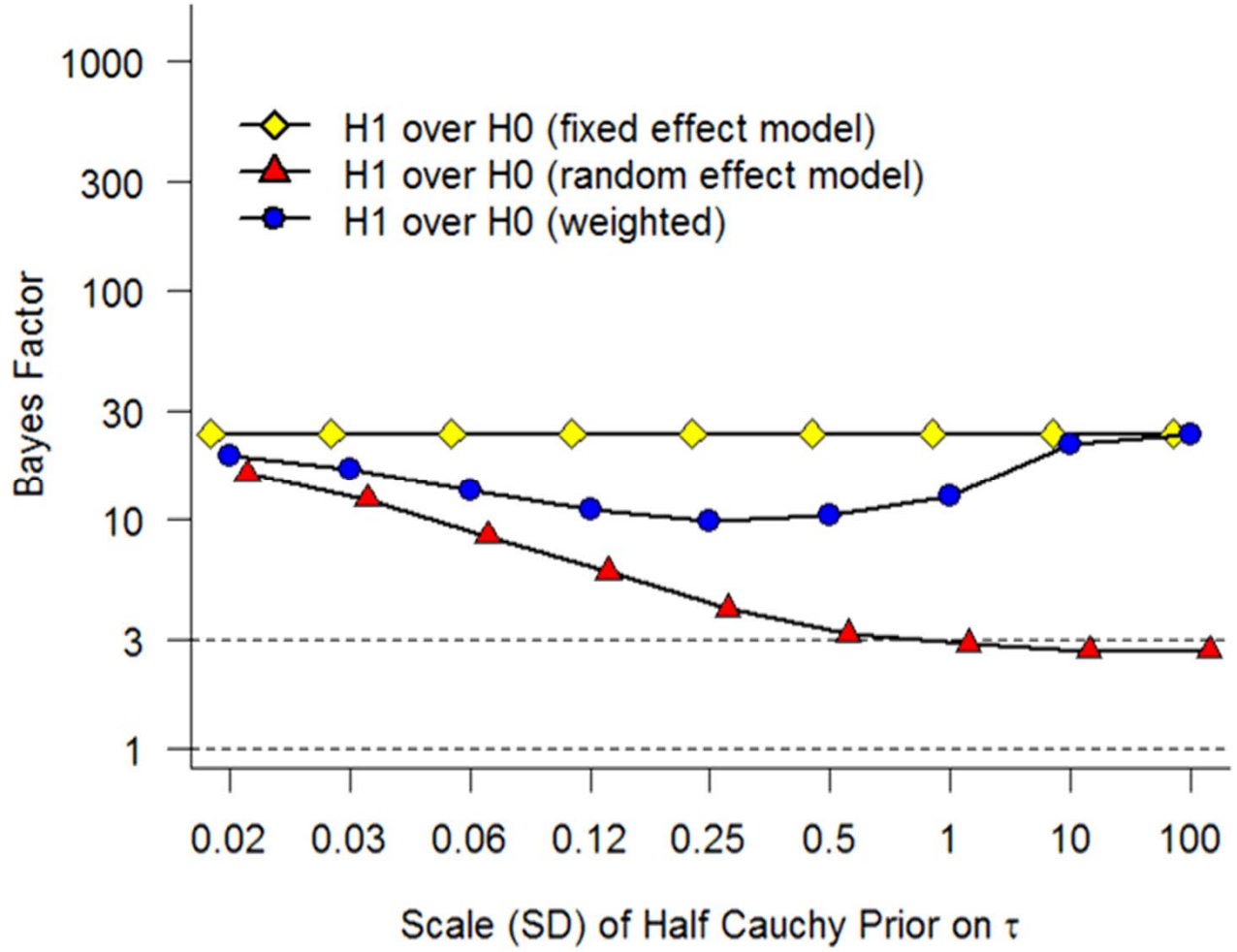
Resolution: Bayesian Model Averaging



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Scheibehenne, et al. (under review) *Psychological Science*

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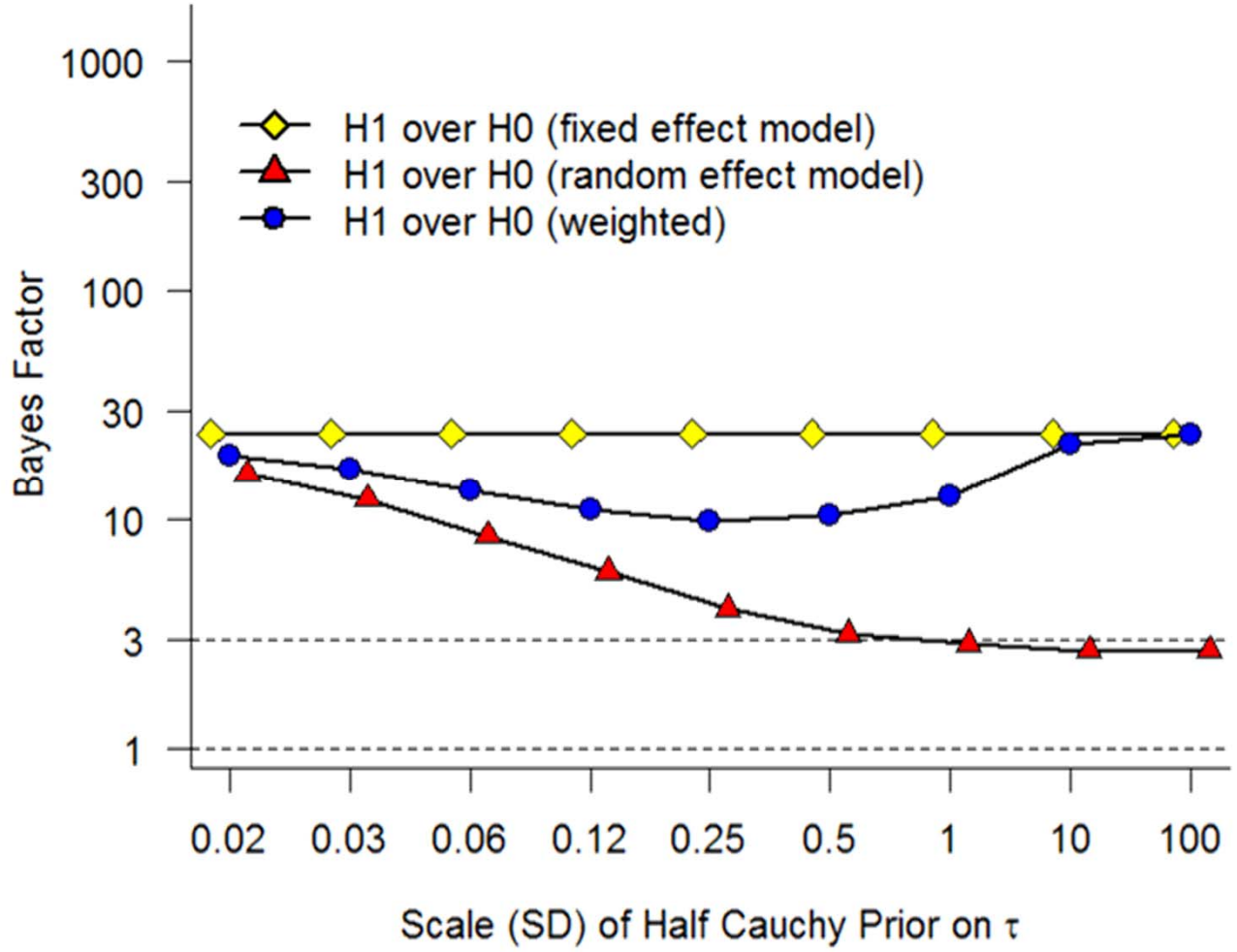

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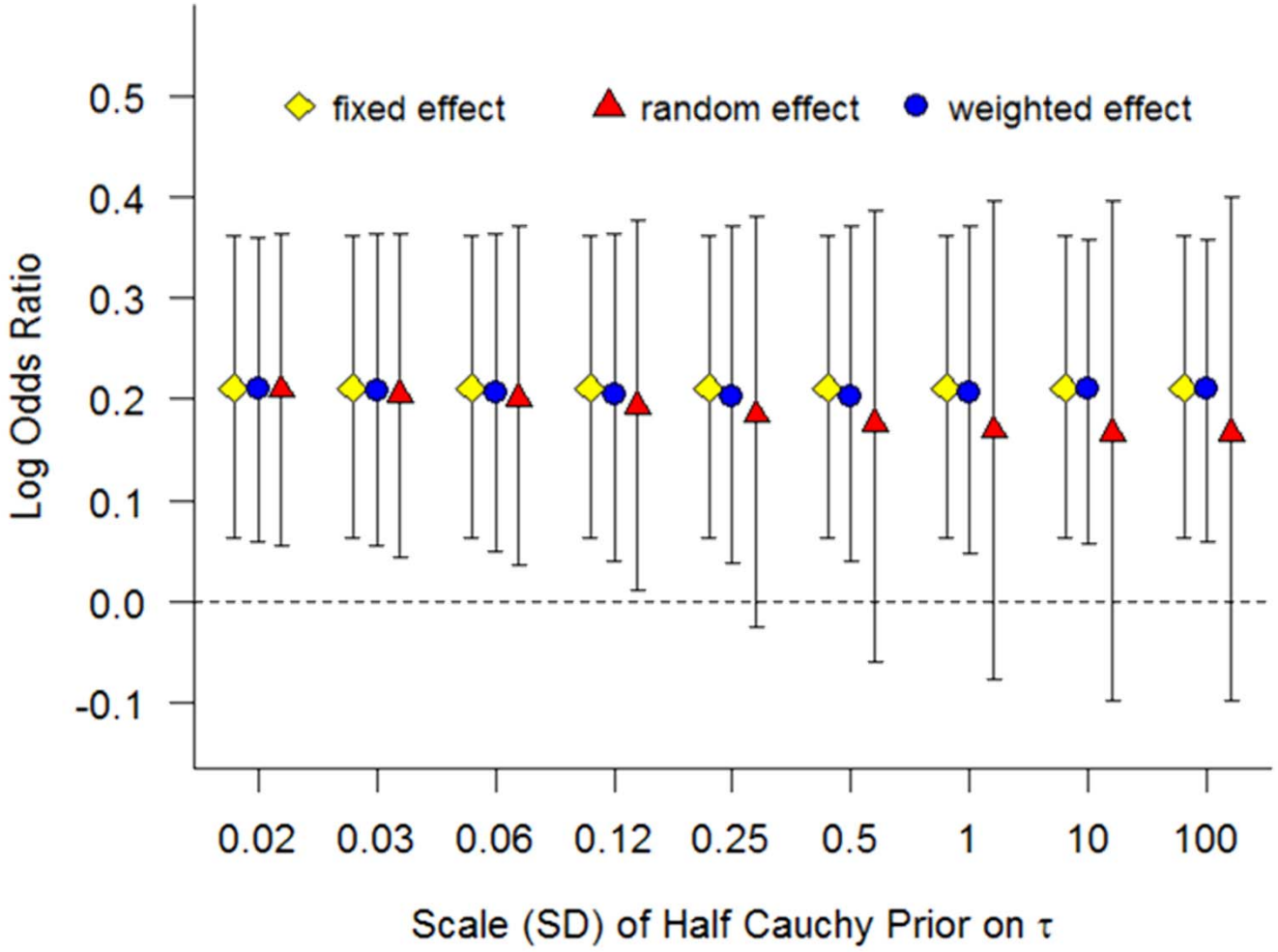

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Broad Application

- Well suited for estimating more complex models
- Lends itself to hierarchical designs

Advantages of Bayesian Approach

- Probabilities for hypotheses
- Able to distinguish b
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- Easy to interpret
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- Possibility to include
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- Model comparison:
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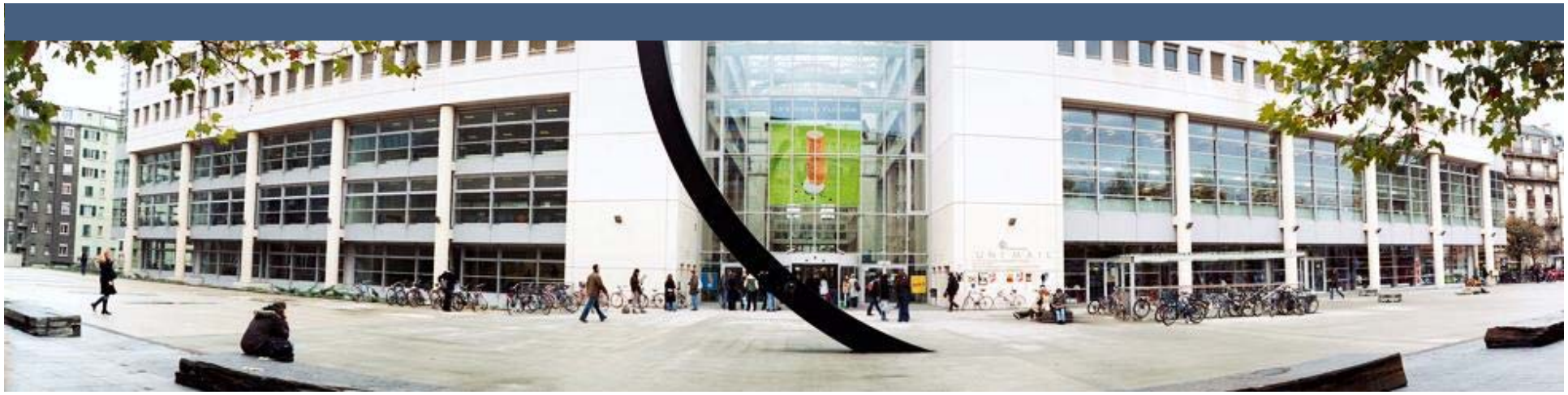

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