Structural equation modeling with R (lavaan package)

Paolo Ghisletta

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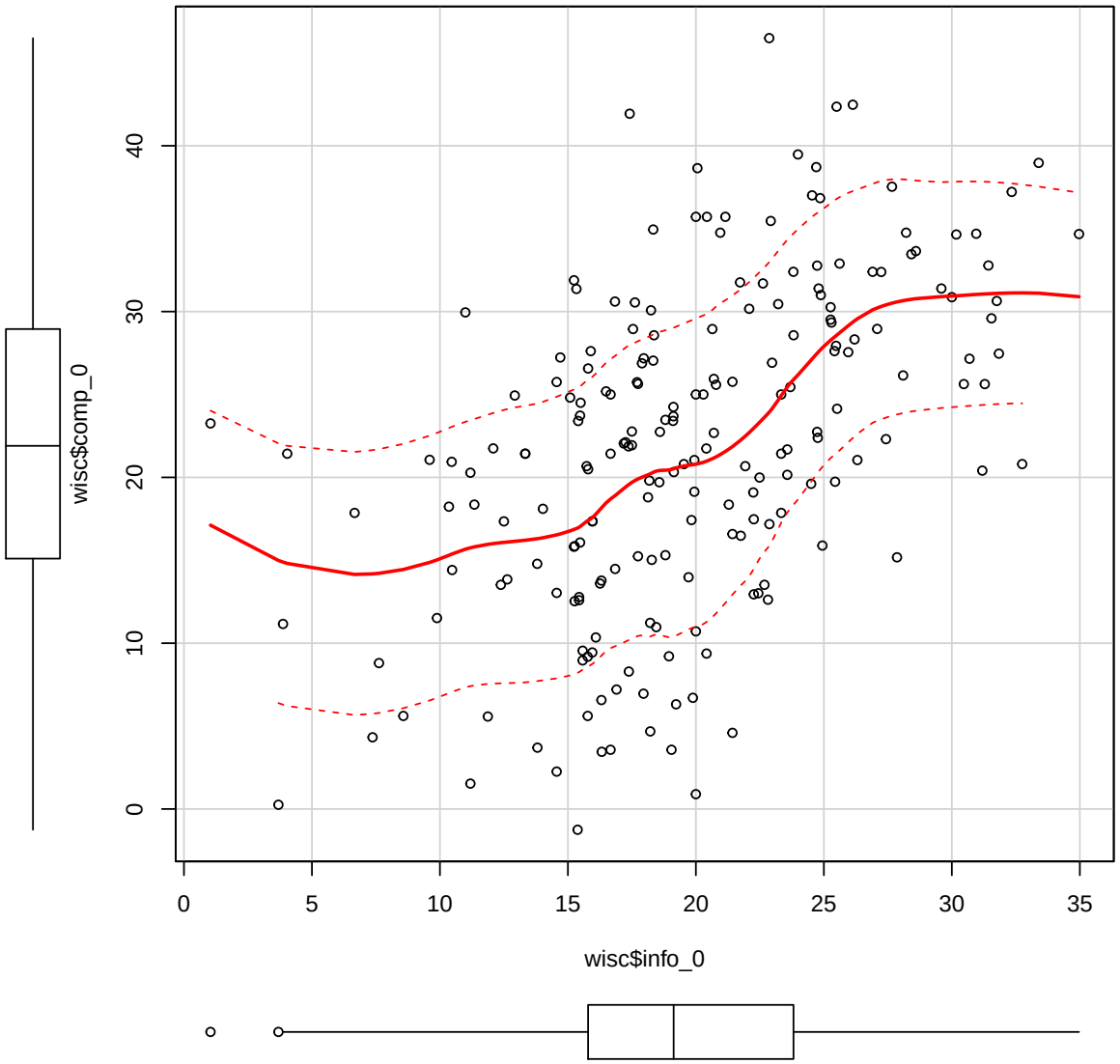
```
# ------------------------------------------------------------------------
# Program: Ghisletta_SEM_R_lavaan_script.R
#  Author: Paolo Ghisletta
# Comment: First SEMs with lavaan
# Comment: Examples modified from
#          http://lavaan.ugent.be/tutorial/tutorial.pdf
# ------------------------------------------------------------------------

### remove any previously created object
rm(list=ls())


# set correct working directory
setwd("C:/PaoloGhisletta/aaaPaolo/congres/2016/use-r_16")

# import wiscraw.sav data from SPSS
# install.packages("foreign", dependencies=T)
library(foreign)
wisc <- read.spss("wisc.sav", use.value.labels="T", to.data.frame="T")
names(wisc)

##  [1] "ID"       "moeducat" "age_06"   "info_0"   "comp_0"   "simi_0"
##  [7] "voca_0"   "picc_0"   "pica_0"   "bloc_0"   "obje_0"   "age_11"
## [13] "info_1"   "comp_1"   "simi_1"   "voca_1"   "picc_1"   "pica_1"
## [19] "bloc_1"   "obje_1"

# install.packages("psych", dependencies=T)
library(psych)
describe(wisc)

##          vars   n   mean    sd median trimmed   mad   min    max  range
## ID          1 204 102.50 59.03 102.50  102.50 75.61  1.00 204.00 203.00
## moeducat    2 204   0.85  0.76   1.00    0.82  1.48  0.00   2.00   2.00
## age_06      3 204   6.07  0.32   6.08    6.06  0.37  5.50   7.33   1.83
## info_0      4 204  19.78  6.12  19.13   19.77  5.53  1.04  34.97  33.93
## comp_0      5 204  21.80  9.74  21.90   21.98 10.32 -1.25  46.49  47.74
## simi_0      6 204  14.90  7.56  14.54   14.57  7.83 -4.52  37.76  42.28
## voca_0      7 204  20.40  6.29  19.84   20.11  6.52  2.77  39.66  36.89
## picc_0      8 204  28.25 12.15  29.20   28.60 12.68 -3.42  55.18  58.60
## pica_0      9 204   9.31  8.41   7.40    7.97  5.77 -1.85  54.30  56.14
## bloc_0     10 204   6.94  6.60   5.81    6.12  4.62 -5.58  42.19  47.78
## obje_0     11 204  25.26 16.39  22.34   23.89 15.60 -4.54  71.10  75.64
## age_11     12 204  10.79  0.31  10.83   10.79  0.37 10.17  11.58   1.42
## info_1     13 204  48.51 12.79  46.34   47.80 13.27 24.76  80.00  55.24
## comp_1     14 204  45.17 12.97  44.53   45.22 12.32  0.19  86.65  86.47
## simi_1     15 204  41.30 14.52  39.44   40.68 14.42 14.29  77.96  63.67
## voca_1     16 204  44.45 11.05  43.80   44.43 12.48 18.17  69.87  51.70
```

```
## picc_1     17 204  54.99 14.43  51.77   54.26 15.61 17.41  93.84  76.43
## pica_1     18 204  52.57 14.10  52.11   52.82 12.19  6.96  83.71  76.75
## bloc_1     19 204  36.56 22.00  33.21   34.52 22.66  6.33  94.21  87.88
## obje_1     20 204  65.32 15.67  67.47   66.76 13.27 12.71  95.44  82.73
##            skew kurtosis   se
## ID         0.00    -1.22 4.13
## moeducat   0.25    -1.25 0.05
## age_06     0.28    -0.04 0.02
## info_0    -0.06     0.17 0.43
## comp_0    -0.13    -0.49 0.68
## simi_0     0.39    -0.14 0.53
## voca_0     0.37     0.02 0.44
## picc_0    -0.24    -0.40 0.85
## pica_0     1.97     5.36 0.59
## bloc_0     2.05     6.77 0.46
## obje_0     0.66    -0.24 1.15
## age_11     0.00    -0.86 0.02
## info_1     0.47    -0.65 0.90
## comp_1    -0.01     0.65 0.91
## simi_1     0.39    -0.38 1.02
## voca_1     0.01    -0.63 0.77
## picc_1     0.40    -0.54 1.01
## pica_1    -0.31     0.55 0.99
## bloc_1     0.62    -0.55 1.54
## obje_1    -0.91     0.82 1.10

# ------------------------------------------------------------------------
### simple regression in lm

library(car)

##
## Attaching package:  'car'
## The following object is masked from 'package:psych':
##
##     logit

scatterplot(wisc$info_0, wisc$comp_0, reg.line=T)
```




```
wiscreg1.lm <- lm(comp_0 ~ info_0, data=wisc)
summary(wiscreg1.lm)
```

```
##
## Call:
## lm(formula = comp_0 ~ info_0, data = wisc)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -21.0852  -5.6297   0.8783   5.7582  22.1925
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.78086    1.99588   2.896  0.00419 **
## info_0       0.80986    0.09643   8.398 7.95e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.408 on 202 degrees of freedom
## Multiple R-squared:  0.2588,Adjusted R-squared:  0.2551
```

```
## F-statistic: 70.53 on 1 and 202 DF,  p-value: 7.95e-15

# simple regression as SEM
# install.packages("lavaan",dep=T)
library(lavaan)

## This is lavaan 0.5-22
## lavaan is BETA software!  Please report any bugs.

# specify model
reg1.model <- 'comp_0 ~ info_0
               comp_0 ~~ comp_0'
# test model
reg1.fit <- sem(reg1.model, data=wisc, meanstructure=T)

## Found more than one class "Model" in cache; using the first, from namespace 'MatrixModels'

# inspect results
reg1.fit

## lavaan (0.5-22) converged normally after  14 iterations
##
##   Number of observations                           204
##
##   Estimator                                         ML
##   Minimum Function Test Statistic                0.000
##   Degrees of freedom                                 0

summary(reg1.fit, fit.measures=T, standardized=T, rsquare=T)

## lavaan (0.5-22) converged normally after  14 iterations
##
##   Number of observations                           204
##
##   Estimator                                         ML
##   Minimum Function Test Statistic                0.000
##   Degrees of freedom                                 0
##
## Model test baseline model:
##
##   Minimum Function Test Statistic               61.092
##   Degrees of freedom                                 1
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1381.298
##   Loglikelihood unrestricted model (H1)      -1381.298
##
##   Number of free parameters                          3
##   Akaike (AIC)                                2768.596
##   Bayesian (BIC)                              2778.550
##   Sample-size adjusted Bayesian (BIC)         2769.046
##
## Root Mean Square Error of Approximation:
```

```
##
##   RMSEA                                          0.000
##   90 Percent Confidence Interval          0.000  0.000
##   P-value RMSEA <= 0.05                             NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   comp_0 ~
##     info_0            0.810    0.096    8.440    0.000    0.810    0.509
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .comp_0            5.781    1.986    2.911    0.004    5.781    0.595
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .comp_0           69.995    6.931   10.100    0.000   69.995    0.741
##
## R-Square:
##                    Estimate
##     comp_0            0.259
```

```
# inspect specified parameters
parTable(reg1.fit)
```

```
##   id    lhs op    rhs user group free ustart exo label plabel  start
## 1  1 comp_0  ~ info_0    1     1    1     NA   0         .p1.  0.000
## 2  2 comp_0 ~~ comp_0    1     1    2     NA   0         .p2. 47.217
## 3  3 info_0 ~~ info_0    0     1    0     NA   1         .p3. 37.261
## 4  4 comp_0 ~1           0     1    3     NA   0         .p4. 21.797
## 5  5 info_0 ~1           0     1    0     NA   1         .p5. 19.776
##      est    se
## 1  0.810 0.096
## 2 69.995 6.931
## 3 37.261 0.000
## 4  5.781 1.986
## 5 19.776 0.000
```

```
# see parameters estimated by default, Table 3 p.11 Rosseel 2012 paper.

# to obtain all estimated parameters must specify for exogenous variable
reg2.model <- 'comp_0 ~ info_0
               comp_0 ~~ comp_0
               info_0 ~~ info_0'

reg2.fit <- sem(reg2.model, data=wisc, meanstructure=T)

## Warning in lavaan::lavaan(model = reg2.model, data = wisc, meanstructure = T, :  lavaan
WARNING: syntax contains parameters involving exogenous covariates; switching to fixed.x
= FALSE
```

```
summary(reg2.fit, fit.measures=T, standardized=T, rsquare=T)

## lavaan (0.5-22) converged normally after  17 iterations
##
##   Number of observations                           204
##
##   Estimator                                         ML
##   Minimum Function Test Statistic                0.000
##   Degrees of freedom                                 0
##
## Model test baseline model:
##
##   Minimum Function Test Statistic               61.092
##   Degrees of freedom                                 1
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1381.298
##   Loglikelihood unrestricted model (H1)      -1381.298
##
##   Number of free parameters                          5
##   Akaike (AIC)                                2772.596
##   Bayesian (BIC)                              2789.187
##   Sample-size adjusted Bayesian (BIC)         2773.345
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.000
##   90 Percent Confidence Interval          0.000  0.000
##   P-value RMSEA <= 0.05                             NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   comp_0 ~
##     info_0            0.810    0.096    8.440    0.000    0.810    0.509
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .comp_0            5.781    1.986    2.911    0.004    5.781    0.595
##     info_0           19.776    0.427   46.273    0.000   19.776    3.240
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

```
##    .comp_0           69.995    6.931   10.100    0.000   69.995    0.741
##     info_0           37.261    3.689   10.100    0.000   37.261    1.000
##
## R-Square:
##                    Estimate
##     comp_0            0.259
```

```
parTable(reg2.fit)
```

```
##   id    lhs op    rhs user group free ustart exo label plabel  start
## 1  1 comp_0  ~ info_0    1     1    1     NA   0         .p1.  0.000
## 2  2 comp_0 ~~ comp_0    1     1    2     NA   0         .p2. 47.217
## 3  3 info_0 ~~ info_0    1     1    3     NA   0         .p3. 18.631
## 4  4 comp_0 ~1           0     1    4     NA   0         .p4. 21.797
## 5  5 info_0 ~1           0     1    5     NA   0         .p5. 19.776
##      est    se
## 1  0.810 0.096
## 2 69.995 6.931
## 3 37.261 3.689
## 4  5.781 1.986
## 5 19.776 0.427
```

```
# specify all parameters
reg3.model <- 'comp_0 ~ info_0
               comp_0 ~~ comp_0
               info_0 ~~ info_0
               comp_0 ~1
               info_0 ~1'

reg3.fit <- lavaan(reg3.model, data=wisc, meanstructure=T)

## Warning in lavaan(reg3.model, data = wisc, meanstructure = T): lavaan WARNING: syntax
contains parameters involving exogenous covariates; switching to fixed.x = FALSE

summary(reg3.fit, fit.measures=T, standardized=T, rsquare=T)

## lavaan (0.5-22) converged normally after  17 iterations
##
##   Number of observations                           204
##
##   Estimator                                         ML
##   Minimum Function Test Statistic                0.000
##   Degrees of freedom                                 0
##
## Model test baseline model:
##
##   Minimum Function Test Statistic               61.092
##   Degrees of freedom                                 1
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1381.298
##   Loglikelihood unrestricted model (H1)      -1381.298
```

```
##
##   Number of free parameters                          5
##   Akaike (AIC)                                2772.596
##   Bayesian (BIC)                              2789.187
##   Sample-size adjusted Bayesian (BIC)         2773.345
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.000
##   90 Percent Confidence Interval          0.000  0.000
##   P-value RMSEA <= 0.05                             NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   comp_0 ~
##     info_0            0.810    0.096    8.440    0.000    0.810    0.509
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .comp_0            5.781    1.986    2.911    0.004    5.781    0.595
##     info_0           19.776    0.427   46.273    0.000   19.776    3.240
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .comp_0           69.995    6.931   10.100    0.000   69.995    0.741
##     info_0           37.261    3.689   10.100    0.000   37.261    1.000
##
## R-Square:
##                    Estimate
##     comp_0            0.259
```

```
parTable(reg3.fit)
```

```
##   id    lhs op    rhs user group free ustart exo label plabel  start
## 1  1 comp_0  ~ info_0    1     1    1     NA   0         .p1.  0.000
## 2  2 comp_0 ~~ comp_0    1     1    2     NA   0         .p2. 47.217
## 3  3 info_0 ~~ info_0    1     1    3     NA   0         .p3. 18.631
## 4  4 comp_0 ~1           1     1    4     NA   0         .p4. 21.797
## 5  5 info_0 ~1           1     1    5     NA   0         .p5. 19.776
##      est    se
## 1  0.810 0.096
## 2 69.995 6.931
## 3 37.261 3.689
## 4  5.781 1.986
## 5 19.776 0.427
```

```
# ------------------------------------------------------------------------
### Factor analysis (CFA example, pp. 4-8 of Rosseel's tutorial)

# import data set used in lavaan tutorial to do EFA
```

```
HS39 <- read.dta("c:/data/HS39.dta")
describe(HS39)
```

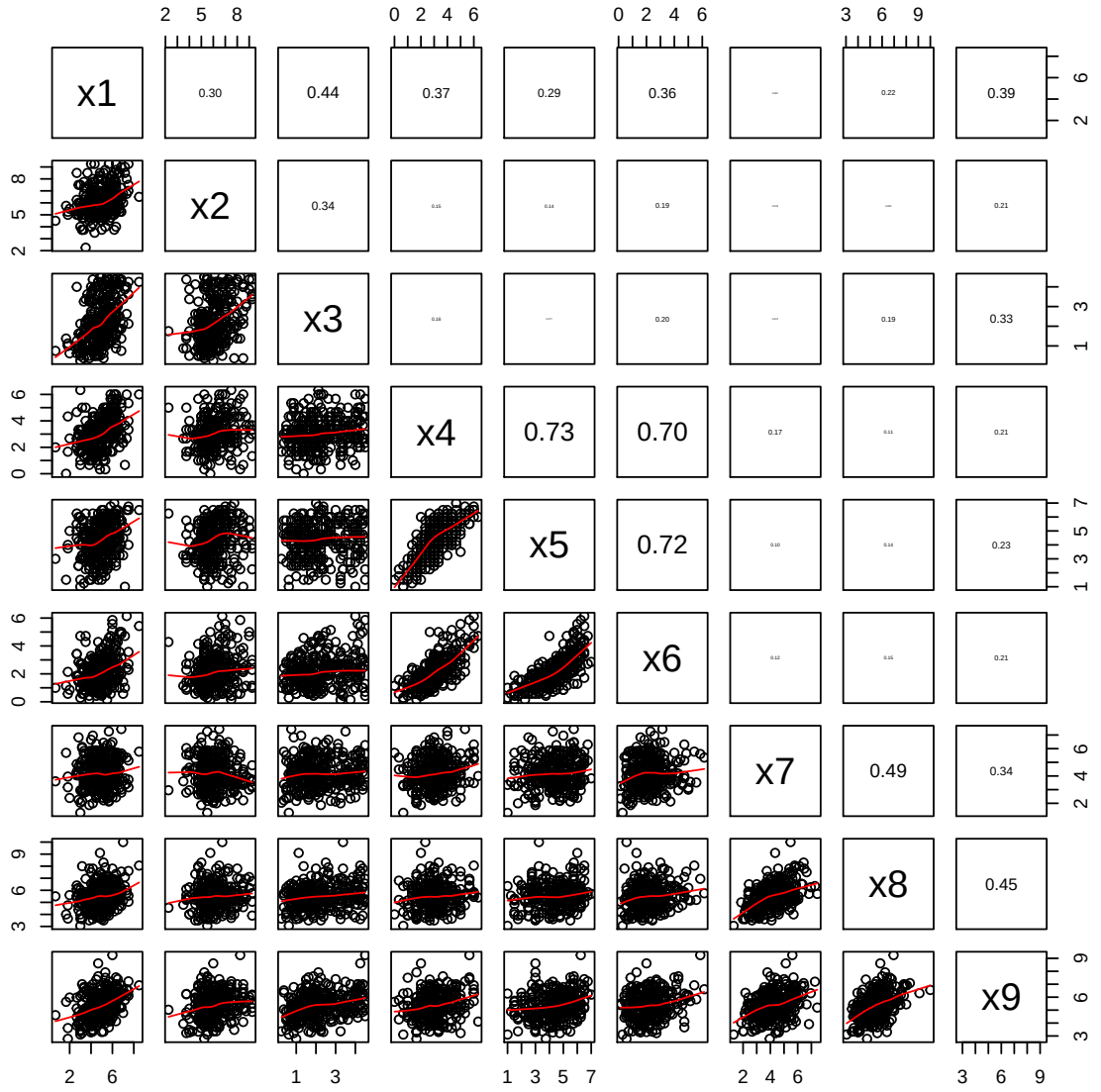
```
##         vars   n   mean     sd median trimmed    mad   min    max  range
## id         1 301 176.55 105.94 163.00  176.78 140.85  1.00 351.00 350.00
## sex        2 301   1.51   0.50   2.00    1.52   0.00  1.00   2.00   1.00
## ageyr      3 301  13.00   1.05  13.00   12.89   1.48 11.00  16.00   5.00
## agemo      4 301   5.38   3.45   5.00    5.32   4.45  0.00  11.00  11.00
## school*    5 301   1.48   0.50   1.00    1.48   0.00  1.00   2.00   1.00
## grade      6 300   7.48   0.50   7.00    7.47   0.00  7.00   8.00   1.00
## x1         7 301   4.94   1.17   5.00    4.96   1.24  0.67   8.50   7.83
## x2         8 301   6.09   1.18   6.00    6.02   1.11  2.25   9.25   7.00
## x3         9 301   2.25   1.13   2.12    2.20   1.30  0.25   4.50   4.25
## x4        10 301   3.06   1.16   3.00    3.02   0.99  0.00   6.33   6.33
## x5        11 301   4.34   1.29   4.50    4.40   1.48  1.00   7.00   6.00
## x6        12 301   2.19   1.10   2.00    2.09   1.06  0.14   6.14   6.00
## x7        13 301   4.19   1.09   4.09    4.16   1.10  1.30   7.43   6.13
## x8        14 301   5.53   1.01   5.50    5.49   0.96  3.05  10.00   6.95
## x9        15 301   5.37   1.01   5.42    5.37   0.99  2.78   9.25   6.47
##           skew kurtosis   se
## id       -0.01    -1.36 6.11
## sex      -0.06    -2.00 0.03
## ageyr     0.69     0.20 0.06
## agemo     0.09    -1.22 0.20
## school*   0.07    -2.00 0.03
## grade     0.09    -2.00 0.03
## x1       -0.25     0.31 0.07
## x2        0.47     0.33 0.07
## x3        0.38    -0.91 0.07
## x4        0.27     0.08 0.07
## x5       -0.35    -0.55 0.07
## x6        0.86     0.82 0.06
## x7        0.25    -0.31 0.06
## x8        0.53     1.17 0.06
## x9        0.20     0.29 0.06
```

```
# limit to cognitive variables x1--x2
HS39cogn <- HS39[,7:15]
describe(HS39cogn)
```

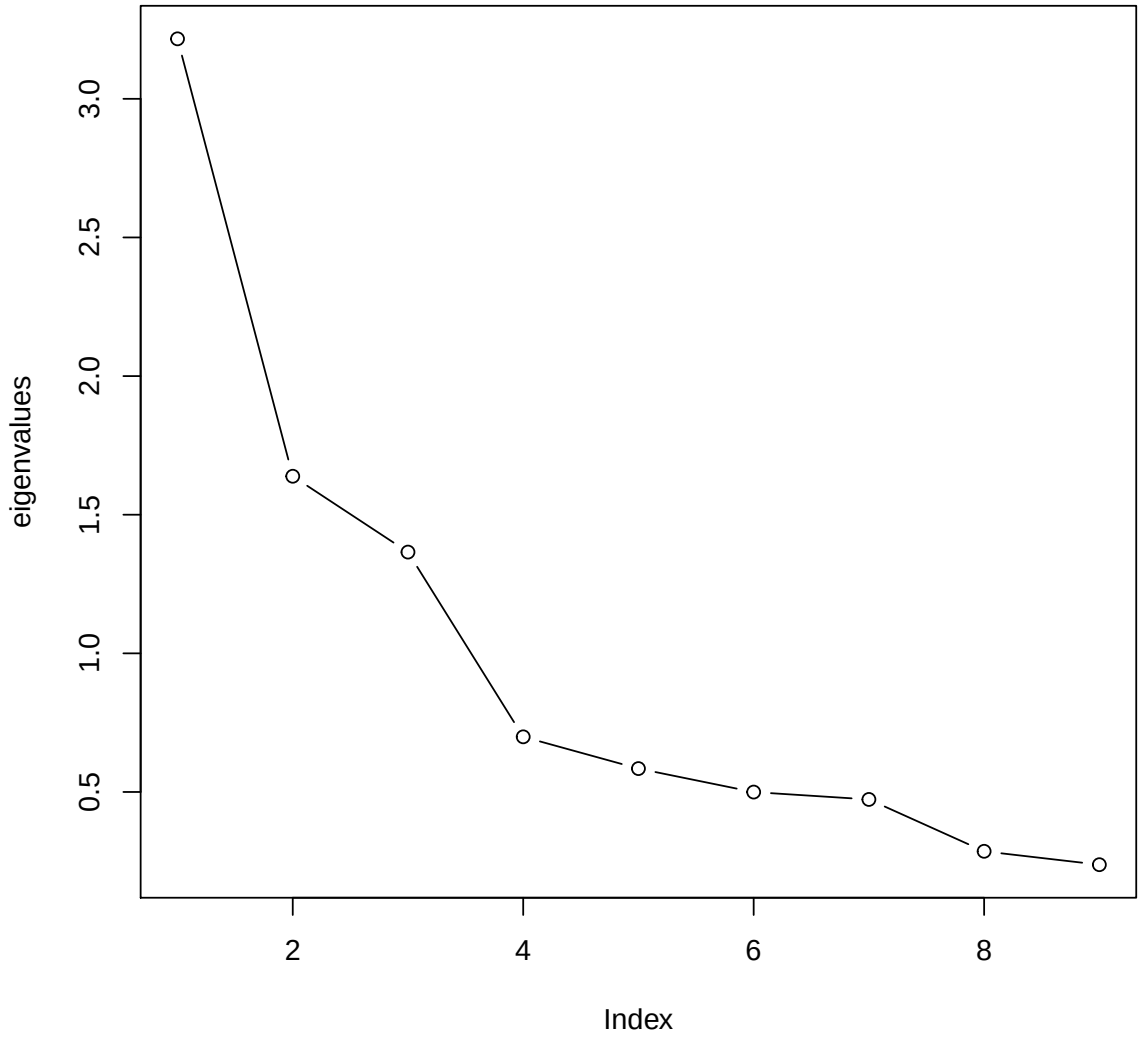
```
##    vars   n mean   sd median trimmed  mad  min   max range  skew kurtosis
## x1    1 301 4.94 1.17   5.00    4.96 1.24 0.67  8.50  7.83 -0.25     0.31
## x2    2 301 6.09 1.18   6.00    6.02 1.11 2.25  9.25  7.00  0.47     0.33
## x3    3 301 2.25 1.13   2.12    2.20 1.30 0.25  4.50  4.25  0.38    -0.91
## x4    4 301 3.06 1.16   3.00    3.02 0.99 0.00  6.33  6.33  0.27     0.08
## x5    5 301 4.34 1.29   4.50    4.40 1.48 1.00  7.00  6.00 -0.35    -0.55
## x6    6 301 2.19 1.10   2.00    2.09 1.06 0.14  6.14  6.00  0.86     0.82
## x7    7 301 4.19 1.09   4.09    4.16 1.10 1.30  7.43  6.13  0.25    -0.31
## x8    8 301 5.53 1.01   5.50    5.49 0.96 3.05 10.00  6.95  0.53     1.17
## x9    9 301 5.37 1.01   5.42    5.37 0.99 2.78  9.25  6.47  0.20     0.29
##      se
## x1 0.07
## x2 0.07
## x3 0.07
## x4 0.07
## x5 0.07
## x6 0.06
## x7 0.06
## x8 0.06
```

```
## x9 0.06

# correlation matrix and p-values
# install.packages("Hmisc", dep=T)
library(Hmisc)

## Loading required package:  lattice
## Loading required package:  survival
## Loading required package:  Formula
## Loading required package:  ggplot2
##
## Attaching package:  'ggplot2'
## The following objects are masked from 'package:psych':
##
##    %+%, alpha
##
## Attaching package:  'Hmisc'
## The following object is masked from 'package:psych':
##
##    describe
## The following objects are masked from 'package:base':
##
##    format.pval, round.POSIXt, trunc.POSIXt, units

HS39cogn.matrix <- as.matrix(HS39cogn)
rcorr(HS39cogn.matrix, type="pearson")

##      x1    x2   x3   x4   x5   x6    x7   x8   x9
## x1 1.00  0.30 0.44 0.37 0.29 0.36  0.07 0.22 0.39
## x2 0.30  1.00 0.34 0.15 0.14 0.19 -0.08 0.09 0.21
## x3 0.44  0.34 1.00 0.16 0.08 0.20  0.07 0.19 0.33
## x4 0.37  0.15 0.16 1.00 0.73 0.70  0.17 0.11 0.21
## x5 0.29  0.14 0.08 0.73 1.00 0.72  0.10 0.14 0.23
## x6 0.36  0.19 0.20 0.70 0.72 1.00  0.12 0.15 0.21
## x7 0.07 -0.08 0.07 0.17 0.10 0.12  1.00 0.49 0.34
## x8 0.22  0.09 0.19 0.11 0.14 0.15  0.49 1.00 0.45
## x9 0.39  0.21 0.33 0.21 0.23 0.21  0.34 0.45 1.00
##
## n= 301
##
##
## P
##    x1     x2     x3     x4     x5     x6     x7     x8     x9
## x1        0.0000 0.0000 0.0000 0.0000 0.0000 0.2475 0.0000 0.0000
## x2 0.0000        0.0000 0.0079 0.0155 0.0008 0.1905 0.1101 0.0003
## x3 0.0000 0.0000        0.0058 0.1816 0.0006 0.2134 0.0012 0.0000
## x4 0.0000 0.0079 0.0058        0.0000 0.0000 0.0025 0.0640 0.0003
## x5 0.0000 0.0155 0.1816 0.0000        0.0000 0.0771 0.0161 0.0000
## x6 0.0000 0.0008 0.0006 0.0000 0.0000        0.0357 0.0093 0.0002
## x7 0.2475 0.1905 0.2134 0.0025 0.0771 0.0357        0.0000 0.0000
## x8 0.0000 0.1101 0.0012 0.0640 0.0161 0.0093 0.0000        0.0000
## x9 0.0000 0.0003 0.0000 0.0003 0.0000 0.0002 0.0000 0.0000

# plot correlation matrix of cognitive variables
panel.cor <- function(x, y, digits=2, prefix="", cex.cor, ...)
{ usr <- par("usr"); on.exit(par(usr))
  par(usr = c(0, 1, 0, 1))
  r <- abs(cor(x, y))
  txt <- format(c(r, 0.123456789), digits=digits)[1]
```

```
  txt <- paste(prefix, txt, sep="")
  if(missing(cex.cor)) cex.cor <- 0.8/strwidth(txt)
  text(0.5, 0.5, txt, cex = cex.cor * r)
}

pairs(HS39cogn, lower.panel=panel.smooth, upper.panel=panel.cor)
```




```
# get eigenvalues and plot them to screeplot
pca <- principal(HS39cogn)
plot(pca$values, type="b", ylab="eigenvalues")
```




```
# efa
#correlation matrix
HS39cogn.cor <- cor(HS39cogn, use="pairwise.complete.obs", method="pearson")
# run efa
EFA3 <- fa(HS39cogn.cor, nfactors=3, n.obs=301, rotate="promax")
# look at loadings
unclass(loadings(EFA3))
```

```
##             MR1         MR3          MR2
## x1  0.145673781  0.62391861  0.008710073
## x2  0.006866091  0.52826214 -0.136580127
## x3 -0.122060306  0.71610286 -0.001634555
## x4  0.840999748  0.01828656  0.002124350
## x5  0.895580862 -0.07467311  0.007462242
## x6  0.803974055  0.07683720 -0.016270961
## x7  0.047179612 -0.17742426  0.736856463
## x8 -0.048379705  0.08953483  0.705632663
## x9  0.002115905  0.36775125  0.454920188
```

```
# look at entire solution
```

```
EFA3
```

```
## Factor Analysis using method =  minres
## Call: fa(r = HS39cogn.cor, nfactors = 3, n.obs = 301, rotate = "promax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##      MR1   MR3   MR2   h2   u2 com
## x1  0.15  0.62  0.01 0.49 0.51 1.1
## x2  0.01  0.53 -0.14 0.25 0.75 1.1
## x3 -0.12  0.72  0.00 0.46 0.54 1.1
## x4  0.84  0.02  0.00 0.72 0.28 1.0
## x5  0.90 -0.07  0.01 0.76 0.24 1.0
## x6  0.80  0.08 -0.02 0.69 0.31 1.0
## x7  0.05 -0.18  0.74 0.50 0.50 1.1
## x8 -0.05  0.09  0.71 0.53 0.47 1.0
## x9  0.00  0.37  0.45 0.46 0.54 1.9
##
##                        MR1  MR3  MR2
## SS loadings           2.20 1.38 1.28
## Proportion Var        0.24 0.15 0.14
## Cumulative Var        0.24 0.40 0.54
## Proportion Explained  0.45 0.28 0.26
## Cumulative Proportion 0.45 0.74 1.00
##
##  With factor correlations of
##      MR1  MR3  MR2
## MR1 1.00 0.40 0.24
## MR3 0.40 1.00 0.34
## MR2 0.24 0.34 1.00
##
## Mean item complexity =  1.2
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  3.05 with Chi S
## The degrees of freedom for the model are 12  and the objective function was  0.08
##
## The root mean square of the residuals (RMSR) is  0.02
## The df corrected root mean square of the residuals is  0.03
##
## The harmonic number of observations is  301 with the empirical chi square  8.03  with prob <  0.78
## The total number of observations was  301  with MLE Chi Square =  22.38  with prob <  0.034
##
## Tucker Lewis Index of factoring reliability =  0.964
## RMSEA index =  0.055  and the 90 % confidence intervals are  0.015 0.088
## BIC =  -46.11
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                MR1  MR3  MR2
## Correlation of scores with factors            0.94 0.85 0.85
## Multiple R square of scores with factors      0.89 0.73 0.73
## Minimum correlation of possible factor scores  0.78 0.46 0.46
```

```
# run CFA with lavaan
# specify model with default specifications of "cfa" syntax
# from p.9 of tutorial: "

HS1.model <- 'visual =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed =~ x7 + x8 + x9
             '
```

```r
# test model
HS1.fit <- cfa (HS1.model, data=HolzingerSwineford1939)

# inspect results
HS1.fit
```

```
## lavaan (0.5-22) converged normally after  35 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic               85.306
##   Degrees of freedom                                24
##   P-value (Chi-square)                           0.000
```

```r
summary(HS1.fit, fit.measures=T, standardized=T, rsquare=T)
```

```
## lavaan (0.5-22) converged normally after  35 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic               85.306
##   Degrees of freedom                                24
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.931
##   Tucker-Lewis Index (TLI)                       0.896
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3737.745
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         21
##   Akaike (AIC)                                7517.490
##   Bayesian (BIC)                              7595.339
##   Sample-size adjusted Bayesian (BIC)         7528.739
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.092
##   90 Percent Confidence Interval          0.071  0.114
##   P-value RMSEA <= 0.05                          0.001
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.065
##
## Parameter Estimates:
```

```
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.900    0.772
##     x2                0.554    0.100    5.554    0.000    0.498    0.424
##     x3                0.729    0.109    6.685    0.000    0.656    0.581
##   textual =~
##     x4                1.000                               0.990    0.852
##     x5                1.113    0.065   17.014    0.000    1.102    0.855
##     x6                0.926    0.055   16.703    0.000    0.917    0.838
##   speed =~
##     x7                1.000                               0.619    0.570
##     x8                1.180    0.165    7.152    0.000    0.731    0.723
##     x9                1.082    0.151    7.155    0.000    0.670    0.665
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.408    0.074    5.552    0.000    0.459    0.459
##     speed             0.262    0.056    4.660    0.000    0.471    0.471
##   textual ~~
##     speed             0.173    0.049    3.518    0.000    0.283    0.283
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.549    0.114    4.833    0.000    0.549    0.404
##    .x2                1.134    0.102   11.146    0.000    1.134    0.821
##    .x3                0.844    0.091    9.317    0.000    0.844    0.662
##    .x4                0.371    0.048    7.779    0.000    0.371    0.275
##    .x5                0.446    0.058    7.642    0.000    0.446    0.269
##    .x6                0.356    0.043    8.277    0.000    0.356    0.298
##    .x7                0.799    0.081    9.823    0.000    0.799    0.676
##    .x8                0.488    0.074    6.573    0.000    0.488    0.477
##    .x9                0.566    0.071    8.003    0.000    0.566    0.558
##     visual            0.809    0.145    5.564    0.000    1.000    1.000
##     textual           0.979    0.112    8.737    0.000    1.000    1.000
##     speed             0.384    0.086    4.451    0.000    1.000    1.000
##
## R-Square:
##                   Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442
```

```
# look at all parameters
parTable(HS1.fit)
```

```
##    id     lhs op     rhs user group free ustart exo label plabel start
## 1   1  visual =~      x1    1     1    0      1   0         .p1. 1.000
```

```
## 2   2  visual =~      x2    1     1     1     NA   0         .p2. 0.778
## 3   3  visual =~      x3    1     1     2     NA   0         .p3. 1.107
## 4   4 textual =~      x4    1     1     0      1   0         .p4. 1.000
## 5   5 textual =~      x5    1     1     3     NA   0         .p5. 1.133
## 6   6 textual =~      x6    1     1     4     NA   0         .p6. 0.924
## 7   7   speed =~      x7    1     1     0      1   0         .p7. 1.000
## 8   8   speed =~      x8    1     1     5     NA   0         .p8. 1.225
## 9   9   speed =~      x9    1     1     6     NA   0         .p9. 0.854
## 10 10      x1 ~~      x1    0     1     7     NA   0        .p10. 0.679
## 11 11      x2 ~~      x2    0     1     8     NA   0        .p11. 0.691
## 12 12      x3 ~~      x3    0     1     9     NA   0        .p12. 0.637
## 13 13      x4 ~~      x4    0     1    10     NA   0        .p13. 0.675
## 14 14      x5 ~~      x5    0     1    11     NA   0        .p14. 0.830
## 15 15      x6 ~~      x6    0     1    12     NA   0        .p15. 0.598
## 16 16      x7 ~~      x7    0     1    13     NA   0        .p16. 0.592
## 17 17      x8 ~~      x8    0     1    14     NA   0        .p17. 0.511
## 18 18      x9 ~~      x9    0     1    15     NA   0        .p18. 0.508
## 19 19  visual ~~  visual    0     1    16     NA   0        .p19. 0.050
## 20 20 textual ~~ textual    0     1    17     NA   0        .p20. 0.050
## 21 21   speed ~~   speed    0     1    18     NA   0        .p21. 0.050
## 22 22  visual ~~ textual    0     1    19     NA   0        .p22. 0.000
## 23 23  visual ~~   speed    0     1    20     NA   0        .p23. 0.000
## 24 24 textual ~~   speed    0     1    21     NA   0        .p24. 0.000
##      est    se
## 1  1.000 0.000
## 2  0.554 0.100
## 3  0.729 0.109
## 4  1.000 0.000
## 5  1.113 0.065
## 6  0.926 0.055
## 7  1.000 0.000
## 8  1.180 0.165
## 9  1.082 0.151
## 10 0.549 0.114
## 11 1.134 0.102
## 12 0.844 0.091
## 13 0.371 0.048
## 14 0.446 0.058
## 15 0.356 0.043
## 16 0.799 0.081
## 17 0.488 0.074
## 18 0.566 0.071
## 19 0.809 0.145
## 20 0.979 0.112
## 21 0.384 0.086
## 22 0.408 0.074
## 23 0.262 0.056
## 24 0.173 0.049

# respecify model without any default specifications
HSfull.model <- 'visual =~ 1*x1 + x2 + x3
                 textual =~ 1*x4 + x5 + x6
                 speed =~ 1*x7 + x8 + x9

                 x1 ~~ x1
                 x2 ~~ x2
                 x3 ~~ x3
                 x4 ~~ x4
```

```
                  x5 ~~ x5
                  x6 ~~ x6
                  x7 ~~ x7
                  x8 ~~ x8
                  x9 ~~ x9

                  visual ~~ visual
                  textual ~~ textual
                  speed ~~ speed

                  visual ~~ textual + speed
                  textual ~~ speed
                 '
HSfull.fit <- lavaan(HSfull.model, data=HolzingerSwineford1939)

summary(HSfull.fit, fit.measures=T, standardized=T, rsquare=T)
```

```
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.900    0.772
##     x2                0.554    0.100    5.554    0.000    0.498    0.424
##     x3                0.729    0.109    6.685    0.000    0.656    0.581
##   textual =~
##     x4                1.000                               0.990    0.852
##     x5                1.113    0.065   17.014    0.000    1.102    0.855
##     x6                0.926    0.055   16.703    0.000    0.917    0.838
##   speed =~
##     x7                1.000                               0.619    0.570
##     x8                1.180    0.165    7.152    0.000    0.731    0.723
##     x9                1.082    0.151    7.155    0.000    0.670    0.665
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.408    0.074    5.552    0.000    0.459    0.459
##     speed             0.262    0.056    4.660    0.000    0.471    0.471
##   textual ~~
##     speed             0.173    0.049    3.518    0.000    0.283    0.283
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.549    0.114    4.833    0.000    0.549    0.404
##    .x2                1.134    0.102   11.146    0.000    1.134    0.821
##    .x3                0.844    0.091    9.317    0.000    0.844    0.662
##    .x4                0.371    0.048    7.779    0.000    0.371    0.275
##    .x5                0.446    0.058    7.642    0.000    0.446    0.269
##    .x6                0.356    0.043    8.277    0.000    0.356    0.298
##    .x7                0.799    0.081    9.823    0.000    0.799    0.676
##    .x8                0.488    0.074    6.573    0.000    0.488    0.477
##    .x9                0.566    0.071    8.003    0.000    0.566    0.558
##     visual            0.809    0.145    5.564    0.000    1.000    1.000
##     textual           0.979    0.112    8.737    0.000    1.000    1.000
##     speed             0.384    0.086    4.451    0.000    1.000    1.000
##
## R-Square:
##                    Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442
```

```
# compare the fits of the two models
anova(HS1.fit, HSfull.fit)
```

```
## Chi Square Difference Test
##
```

```
##            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HS1.fit    24 7517.5 7595.3 85.305
## HSfull.fit 24 7517.5 7595.3 85.305          0       0          1
```

```
# respecify model with different identification scaling
HS2.model <- 'visual =~ NA*x1 + x2 + x3
              textual =~ NA*x4 + x5 + x6
              speed =~ NA*x7 + x8 + x9
              '

HS2.fit <- cfa (HS2.model, data=HolzingerSwineford1939, std.lv=T)

summary(HS2.fit, fit.measures=T, standardized=T, rsquare=T)
```

```
## lavaan (0.5-22) converged normally after  22 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic               85.306
##   Degrees of freedom                                24
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.931
##   Tucker-Lewis Index (TLI)                       0.896
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3737.745
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         21
##   Akaike (AIC)                                7517.490
##   Bayesian (BIC)                              7595.339
##   Sample-size adjusted Bayesian (BIC)         7528.739
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.092
##   90 Percent Confidence Interval          0.071  0.114
##   P-value RMSEA <= 0.05                          0.001
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.065
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
```

```
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1               0.900    0.081   11.127    0.000    0.900    0.772
##     x2               0.498    0.077    6.429    0.000    0.498    0.424
##     x3               0.656    0.074    8.817    0.000    0.656    0.581
##   textual =~
##     x4               0.990    0.057   17.474    0.000    0.990    0.852
##     x5               1.102    0.063   17.576    0.000    1.102    0.855
##     x6               0.917    0.054   17.082    0.000    0.917    0.838
##   speed =~
##     x7               0.619    0.070    8.903    0.000    0.619    0.570
##     x8               0.731    0.066   11.090    0.000    0.731    0.723
##     x9               0.670    0.065   10.305    0.000    0.670    0.665
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual          0.459    0.064    7.189    0.000    0.459    0.459
##     speed            0.471    0.073    6.461    0.000    0.471    0.471
##   textual ~~
##     speed            0.283    0.069    4.117    0.000    0.283    0.283
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1               0.549    0.114    4.833    0.000    0.549    0.404
##    .x2               1.134    0.102   11.146    0.000    1.134    0.821
##    .x3               0.844    0.091    9.317    0.000    0.844    0.662
##    .x4               0.371    0.048    7.778    0.000    0.371    0.275
##    .x5               0.446    0.058    7.642    0.000    0.446    0.269
##    .x6               0.356    0.043    8.277    0.000    0.356    0.298
##    .x7               0.799    0.081    9.823    0.000    0.799    0.676
##    .x8               0.488    0.074    6.573    0.000    0.488    0.477
##    .x9               0.566    0.071    8.003    0.000    0.566    0.558
##     visual           1.000                               1.000    1.000
##     textual          1.000                               1.000    1.000
##     speed            1.000                               1.000    1.000
##
## R-Square:
##                   Estimate
##     x1               0.596
##     x2               0.179
##     x3               0.338
##     x4               0.725
##     x5               0.731
##     x6               0.702
##     x7               0.324
##     x8               0.523
##     x9               0.442
```

```
anova(HS1.fit, HS2.fit)
```

```
## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HS1.fit 24 7517.5 7595.3 85.305
## HS2.fit 24 7517.5 7595.3 85.305 3.8017e-10       0  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# even simpler: std.lv=T overwrites default of first loading equal to 1
HS3.fit <- cfa (HS1.model, data=HolzingerSwineford1939, std.lv=T)

summary(HS3.fit, fit.measures=T, standardized=T, rsquare=T)
```

```
## lavaan (0.5-22) converged normally after  22 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic               85.306
##   Degrees of freedom                                24
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.931
##   Tucker-Lewis Index (TLI)                       0.896
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3737.745
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         21
##   Akaike (AIC)                                7517.490
##   Bayesian (BIC)                              7595.339
##   Sample-size adjusted Bayesian (BIC)         7528.739
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.092
##   90 Percent Confidence Interval          0.071  0.114
##   P-value RMSEA <= 0.05                          0.001
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.065
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                0.900    0.081   11.127    0.000    0.900    0.772
##     x2                0.498    0.077    6.429    0.000    0.498    0.424
##     x3                0.656    0.074    8.817    0.000    0.656    0.581
##   textual =~
##     x4                0.990    0.057   17.474    0.000    0.990    0.852
```

```
##     x5                1.102    0.063   17.576    0.000    1.102    0.855
##     x6                0.917    0.054   17.082    0.000    0.917    0.838
##   speed =~
##     x7                0.619    0.070    8.903    0.000    0.619    0.570
##     x8                0.731    0.066   11.090    0.000    0.731    0.723
##     x9                0.670    0.065   10.305    0.000    0.670    0.665
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.459    0.064    7.189    0.000    0.459    0.459
##     speed             0.471    0.073    6.461    0.000    0.471    0.471
##   textual ~~
##     speed             0.283    0.069    4.117    0.000    0.283    0.283
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.549    0.114    4.833    0.000    0.549    0.404
##    .x2                1.134    0.102   11.146    0.000    1.134    0.821
##    .x3                0.844    0.091    9.317    0.000    0.844    0.662
##    .x4                0.371    0.048    7.778    0.000    0.371    0.275
##    .x5                0.446    0.058    7.642    0.000    0.446    0.269
##    .x6                0.356    0.043    8.277    0.000    0.356    0.298
##    .x7                0.799    0.081    9.823    0.000    0.799    0.676
##    .x8                0.488    0.074    6.573    0.000    0.488    0.477
##    .x9                0.566    0.071    8.003    0.000    0.566    0.558
##     visual            1.000                               1.000    1.000
##     textual           1.000                               1.000    1.000
##     speed             1.000                               1.000    1.000
##
## R-Square:
##                   Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442
```

```
anova(HS1.fit, HS3.fit)
```

```
## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HS1.fit 24 7517.5 7595.3 85.305
## HS3.fit 24 7517.5 7595.3 85.305 3.8017e-10       0  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# respecify model with orthogonal factors
HS4.model <- 'visual =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed =~ x7 + x8 + x9
              visual ~~ 0*textual + 0*speed
              textual ~~ 0*speed
              '
```

```
HS4.fit <- cfa (HS4.model, data=HolzingerSwineford1939)

summary(HS4.fit, fit.measures=T, standardized=T, rsquare=T)

## lavaan (0.5-22) converged normally after  32 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic              153.527
##   Degrees of freedom                                27
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.857
##   Tucker-Lewis Index (TLI)                       0.809
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3771.856
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         18
##   Akaike (AIC)                                7579.711
##   Bayesian (BIC)                              7646.439
##   Sample-size adjusted Bayesian (BIC)         7589.354
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.125
##   90 Percent Confidence Interval          0.106  0.144
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.161
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.724    0.621
##     x2                0.778    0.141    5.532    0.000    0.563    0.479
##     x3                1.107    0.214    5.173    0.000    0.801    0.710
##   textual =~
##     x4                1.000                               0.984    0.847
##     x5                1.133    0.067   16.906    0.000    1.115    0.866
```

```
##     x6                0.924    0.056   16.391    0.000    0.910    0.832
##   speed =~
##     x7                1.000                             0.661    0.608
##     x8                1.225    0.190    6.460    0.000    0.810    0.801
##     x9                0.854    0.121    7.046    0.000    0.565    0.561
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.000                             0.000    0.000
##     speed             0.000                             0.000    0.000
##   textual ~~
##     speed             0.000                             0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.835    0.118    7.064    0.000    0.835    0.614
##    .x2                1.065    0.105   10.177    0.000    1.065    0.771
##    .x3                0.633    0.129    4.899    0.000    0.633    0.496
##    .x4                0.382    0.049    7.805    0.000    0.382    0.283
##    .x5                0.416    0.059    7.038    0.000    0.416    0.251
##    .x6                0.369    0.044    8.367    0.000    0.369    0.308
##    .x7                0.746    0.086    8.650    0.000    0.746    0.631
##    .x8                0.366    0.097    3.794    0.000    0.366    0.358
##    .x9                0.696    0.072    9.640    0.000    0.696    0.686
##     visual            0.524    0.130    4.021    0.000    1.000    1.000
##     textual           0.969    0.112    8.640    0.000    1.000    1.000
##     speed             0.437    0.097    4.520    0.000    1.000    1.000
##
## R-Square:
##                   Estimate
##     x1                0.386
##     x2                0.229
##     x3                0.504
##     x4                0.717
##     x5                0.749
##     x6                0.692
##     x7                0.369
##     x8                0.642
##     x9                0.314

# compare fit of oblique vs. orthogonal factors
anova(HS1.fit, HS4.fit)

## Chi Square Difference Test
##
##         Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## HS1.fit 24 7517.5 7595.3  85.305
## HS4.fit 27 7579.7 7646.4 153.527     68.222       3  1.026e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# simpler with orthogonal option
HS5.fit <- cfa (HS1.model, data=HolzingerSwineford1939, orthogonal=T)

summary(HS5.fit, fit.measures=T, standardized=T, rsquare=T)

## lavaan (0.5-22) converged normally after  32 iterations
##
```

```
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic              153.527
##   Degrees of freedom                                27
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.857
##   Tucker-Lewis Index (TLI)                       0.809
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3771.856
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         18
##   Akaike (AIC)                                7579.711
##   Bayesian (BIC)                              7646.439
##   Sample-size adjusted Bayesian (BIC)         7589.354
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.125
##   90 Percent Confidence Interval          0.106  0.144
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.161
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.724    0.621
##     x2                0.778    0.141    5.532    0.000    0.563    0.479
##     x3                1.107    0.214    5.173    0.000    0.801    0.710
##   textual =~
##     x4                1.000                               0.984    0.847
##     x5                1.133    0.067   16.906    0.000    1.115    0.866
##     x6                0.924    0.056   16.391    0.000    0.910    0.832
##   speed =~
##     x7                1.000                               0.661    0.608
##     x8                1.225    0.190    6.460    0.000    0.810    0.801
##     x9                0.854    0.121    7.046    0.000    0.565    0.561
##
```

```
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.000                               0.000    0.000
##     speed             0.000                               0.000    0.000
##   textual ~~
##     speed             0.000                               0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.835    0.118    7.064    0.000    0.835    0.614
##    .x2                1.065    0.105   10.177    0.000    1.065    0.771
##    .x3                0.633    0.129    4.899    0.000    0.633    0.496
##    .x4                0.382    0.049    7.805    0.000    0.382    0.283
##    .x5                0.416    0.059    7.038    0.000    0.416    0.251
##    .x6                0.369    0.044    8.367    0.000    0.369    0.308
##    .x7                0.746    0.086    8.650    0.000    0.746    0.631
##    .x8                0.366    0.097    3.794    0.000    0.366    0.358
##    .x9                0.696    0.072    9.640    0.000    0.696    0.686
##     visual            0.524    0.130    4.021    0.000    1.000    1.000
##     textual           0.969    0.112    8.640    0.000    1.000    1.000
##     speed             0.437    0.097    4.520    0.000    1.000    1.000
##
## R-Square:
##                    Estimate
##     x1                0.386
##     x2                0.229
##     x3                0.504
##     x4                0.717
##     x5                0.749
##     x6                0.692
##     x7                0.369
##     x8                0.642
##     x9                0.314

anova(HS4.fit, HS5.fit)

## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HS4.fit 27 7579.7 7646.4 153.53
## HS5.fit 27 7579.7 7646.4 153.53          0       0          1

# use labels and equality constraints
HS6.model <- 'visual =~ x1 + a*x2 + a*x3
              textual =~ x4 + x5 + x6
              speed =~ x7 + x8 + x9
             '

HS6.fit <- cfa (HS6.model, data=HolzingerSwineford1939)

# check equality constraint
coef(HS6.fit)

##                a                a      textual=~x5      textual=~x6
##            0.649            0.649            1.113            0.926
##        speed=~x8        speed=~x9           x1~~x1           x2~~x2
##            1.182            1.075            0.549            1.114
##           x3~~x3           x4~~x4           x5~~x5           x6~~x6
```

```
##            0.877            0.371            0.446            0.356
##           x7~~x7           x8~~x8           x9~~x9   visual~~visual
##            0.798            0.484            0.570            0.810
## textual~~textual     speed~~speed  visual~~textual    visual~~speed
##            0.979            0.385            0.414            0.259
##   textual~~speed
##            0.173
```

```
inspect(HS6.fit)
```

```
##
## Note: model contains equality constraints:
##
##   lhs op rhs
## 1   1 ==   2
##
## $lambda
##    visual textul speed
## x1      0      0     0
## x2      1      0     0
## x3      2      0     0
## x4      0      0     0
## x5      0      3     0
## x6      0      4     0
## x7      0      0     0
## x8      0      0     5
## x9      0      0     6
##
## $theta
##    x1 x2 x3 x4 x5 x6 x7 x8 x9
## x1  7
## x2  0  8
## x3  0  0  9
## x4  0  0  0 10
## x5  0  0  0  0 11
## x6  0  0  0  0  0 12
## x7  0  0  0  0  0  0 13
## x8  0  0  0  0  0  0  0 14
## x9  0  0  0  0  0  0  0  0 15
##
## $psi
##         visual textul speed
## visual  16
## textual 19     17
## speed   20     21     18
```

```
# careful with the inspect result!
parTable(HS6.fit)
```

```
##    id     lhs op     rhs user group free ustart exo label plabel start
## 1   1  visual =~      x1    1     1    0      1   0         .p1. 1.000
## 2   2  visual =~      x2    1     1    1     NA   0     a   .p2. 0.778
## 3   3  visual =~      x3    1     1    2     NA   0     a   .p3. 1.107
## 4   4 textual =~      x4    1     1    0      1   0         .p4. 1.000
## 5   5 textual =~      x5    1     1    3     NA   0         .p5. 1.133
## 6   6 textual =~      x6    1     1    4     NA   0         .p6. 0.924
## 7   7   speed =~      x7    1     1    0      1   0         .p7. 1.000
## 8   8   speed =~      x8    1     1    5     NA   0         .p8. 1.225
## 9   9   speed =~      x9    1     1    6     NA   0         .p9. 0.854
```

```
## 10 10      x1 ~~      x1    0     1    7     NA   0        .p10. 0.679
## 11 11      x2 ~~      x2    0     1    8     NA   0        .p11. 0.691
## 12 12      x3 ~~      x3    0     1    9     NA   0        .p12. 0.637
## 13 13      x4 ~~      x4    0     1   10     NA   0        .p13. 0.675
## 14 14      x5 ~~      x5    0     1   11     NA   0        .p14. 0.830
## 15 15      x6 ~~      x6    0     1   12     NA   0        .p15. 0.598
## 16 16      x7 ~~      x7    0     1   13     NA   0        .p16. 0.592
## 17 17      x8 ~~      x8    0     1   14     NA   0        .p17. 0.511
## 18 18      x9 ~~      x9    0     1   15     NA   0        .p18. 0.508
## 19 19  visual ~~  visual    0     1   16     NA   0        .p19. 0.050
## 20 20 textual ~~ textual    0     1   17     NA   0        .p20. 0.050
## 21 21   speed ~~   speed    0     1   18     NA   0        .p21. 0.050
## 22 22  visual ~~ textual    0     1   19     NA   0        .p22. 0.000
## 23 23  visual ~~   speed    0     1   20     NA   0        .p23. 0.000
## 24 24 textual ~~   speed    0     1   21     NA   0        .p24. 0.000
## 25 25    .p2. ==    .p3.    2     0    0     NA   0              0.000
##      est    se
## 1  1.000 0.000
## 2  0.649 0.088
## 3  0.649 0.088
## 4  1.000 0.000
## 5  1.113 0.065
## 6  0.926 0.055
## 7  1.000 0.000
## 8  1.182 0.165
## 9  1.075 0.150
## 10 0.549 0.114
## 11 1.114 0.103
## 12 0.877 0.085
## 13 0.371 0.048
## 14 0.446 0.058
## 15 0.356 0.043
## 16 0.798 0.081
## 17 0.484 0.075
## 18 0.570 0.071
## 19 0.810 0.146
## 20 0.979 0.112
## 21 0.385 0.086
## 22 0.414 0.074
## 23 0.259 0.056
## 24 0.173 0.049
## 25 0.000 0.000
```

```
summary(HS6.fit, fit.measures=T, standardized=T, rsquare=T)
```

```
## lavaan (0.5-22) converged normally after  36 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic               87.971
##   Degrees of freedom                                25
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
##   P-value                                        0.000
```

```
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.929
##   Tucker-Lewis Index (TLI)                       0.897
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3739.077
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         20
##   Akaike (AIC)                                7518.155
##   Bayesian (BIC)                              7592.297
##   Sample-size adjusted Bayesian (BIC)         7528.868
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.091
##   90 Percent Confidence Interval          0.071  0.113
##   P-value RMSEA <= 0.05                          0.001
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.068
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.900    0.772
##     x2         (a)    0.649    0.088    7.355    0.000    0.584    0.484
##     x3         (a)    0.649    0.088    7.355    0.000    0.584    0.529
##   textual =~
##     x4                1.000                               0.990    0.852
##     x5                1.113    0.065   17.019    0.000    1.102    0.855
##     x6                0.926    0.055   16.705    0.000    0.917    0.838
##   speed =~
##     x7                1.000                               0.621    0.571
##     x8                1.182    0.165    7.150    0.000    0.734    0.726
##     x9                1.075    0.150    7.157    0.000    0.667    0.662
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.414    0.074    5.613    0.000    0.465    0.465
##     speed             0.259    0.056    4.617    0.000    0.464    0.464
##   textual ~~
##     speed             0.173    0.049    3.510    0.000    0.282    0.282
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.549    0.114    4.805    0.000    0.549    0.404
##    .x2                1.114    0.103   10.850    0.000    1.114    0.766
```

```
##    .x3                0.877    0.085   10.341    0.000    0.877    0.720
##    .x4                0.371    0.048    7.783    0.000    0.371    0.275
##    .x5                0.446    0.058    7.644    0.000    0.446    0.269
##    .x6                0.356    0.043    8.281    0.000    0.356    0.298
##    .x7                0.798    0.081    9.801    0.000    0.798    0.674
##    .x8                0.484    0.075    6.490    0.000    0.484    0.473
##    .x9                0.570    0.071    8.056    0.000    0.570    0.561
##     visual            0.810    0.146    5.547    0.000    1.000    1.000
##     textual           0.979    0.112    8.737    0.000    1.000    1.000
##     speed             0.385    0.086    4.459    0.000    1.000    1.000
##
## R-Square:
##                    Estimate
##     x1                0.596
##     x2                0.234
##     x3                0.280
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.326
##     x8                0.527
##     x9                0.439

# compare fit with and without constraint
anova(HS1.fit, HS6.fit)

## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HS1.fit 24 7517.5 7595.3 85.305
## HS6.fit 25 7518.2 7592.3 87.971      2.665       1     0.1026

# ------------------------------------------------------------------
### CFA example with means and intercepts

# reminder of first CFA
HS1.model <- 'visual =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed =~ x7 + x8 + x9
             '

# test model with means and intercepts
HS7.fit <- cfa (HS1.model, data=HolzingerSwineford1939, meanstructure=T)

summary(HS7.fit, fit.measures=T, standardized=T, rsquare=T)

## lavaan (0.5-22) converged normally after  35 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic               85.306
##   Degrees of freedom                                24
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                                36
```

```
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.931
##   Tucker-Lewis Index (TLI)                       0.896
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3737.745
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         30
##   Akaike (AIC)                                7535.490
##   Bayesian (BIC)                              7646.703
##   Sample-size adjusted Bayesian (BIC)         7551.560
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.092
##   90 Percent Confidence Interval          0.071  0.114
##   P-value RMSEA <= 0.05                          0.001
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.060
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.900    0.772
##     x2                0.554    0.100    5.554    0.000    0.498    0.424
##     x3                0.729    0.109    6.685    0.000    0.656    0.581
##   textual =~
##     x4                1.000                               0.990    0.852
##     x5                1.113    0.065   17.014    0.000    1.102    0.855
##     x6                0.926    0.055   16.703    0.000    0.917    0.838
##   speed =~
##     x7                1.000                               0.619    0.570
##     x8                1.180    0.165    7.152    0.000    0.731    0.723
##     x9                1.082    0.151    7.155    0.000    0.670    0.665
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.408    0.074    5.552    0.000    0.459    0.459
##     speed             0.262    0.056    4.660    0.000    0.471    0.471
##   textual ~~
##     speed             0.173    0.049    3.518    0.000    0.283    0.283
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                4.936    0.067   73.473    0.000    4.936    4.235
```

```
##    .x2                6.088    0.068   89.855    0.000    6.088    5.179
##    .x3                2.250    0.065   34.579    0.000    2.250    1.993
##    .x4                3.061    0.067   45.694    0.000    3.061    2.634
##    .x5                4.341    0.074   58.452    0.000    4.341    3.369
##    .x6                2.186    0.063   34.667    0.000    2.186    1.998
##    .x7                4.186    0.063   66.766    0.000    4.186    3.848
##    .x8                5.527    0.058   94.854    0.000    5.527    5.467
##    .x9                5.374    0.058   92.546    0.000    5.374    5.334
##     visual            0.000                             0.000    0.000
##     textual           0.000                             0.000    0.000
##     speed             0.000                             0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.549    0.114    4.833    0.000    0.549    0.404
##    .x2                1.134    0.102   11.146    0.000    1.134    0.821
##    .x3                0.844    0.091    9.317    0.000    0.844    0.662
##    .x4                0.371    0.048    7.779    0.000    0.371    0.275
##    .x5                0.446    0.058    7.642    0.000    0.446    0.269
##    .x6                0.356    0.043    8.277    0.000    0.356    0.298
##    .x7                0.799    0.081    9.823    0.000    0.799    0.676
##    .x8                0.488    0.074    6.573    0.000    0.488    0.477
##    .x9                0.566    0.071    8.003    0.000    0.566    0.558
##     visual            0.809    0.145    5.564    0.000    1.000    1.000
##     textual           0.979    0.112    8.737    0.000    1.000    1.000
##     speed             0.384    0.086    4.451    0.000    1.000    1.000
##
## R-Square:
##                   Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442
```

```
fitMeasures(HS7.fit)
```

```
##                npar                fmin               chisq
##              30.000               0.142              85.306
##                  df              pvalue      baseline.chisq
##              24.000               0.000             918.852
##         baseline.df     baseline.pvalue                 cfi
##              36.000               0.000               0.931
##                 tli                nnfi                 rfi
##               0.896               0.896               0.861
##                 nfi                pnfi                 ifi
##               0.907               0.605               0.931
##                 rni                logl   unrestricted.logl
##               0.931           -3737.745           -3695.092
##                 aic                 bic              ntotal
##            7535.490            7646.703             301.000
##                bic2               rmsea      rmsea.ci.lower
##            7551.560               0.092               0.071
##      rmsea.ci.upper        rmsea.pvalue                 rmr
##               0.114               0.001               0.082
```

```
##          rmr_nomean               srmr        srmr_bentler
##               0.082              0.060               0.060
## srmr_bentler_nomean         srmr_bollen  srmr_bollen_nomean
##               0.065              0.060               0.065
##          srmr_mplus   srmr_mplus_nomean              cn_05
##               0.060              0.065             129.490
##               cn_01                gfi                agfi
##             152.654              0.996               0.991
##                pgfi                mfi                ecvi
##               0.443              0.903                  NA
```

```
# model variables' means as a function of factors' means

HS8.model <- 'visual =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed =~ x7 + x8 + x9

              x1+x2+x3+x4+x5+x6+x7+x8+x9 ~ 0*1
              visual + textual + speed ~ 1
             '

HS8.fit <- cfa (HS8.model, data=HolzingerSwineford1939, meanstructure=T)

summary(HS8.fit, fit.measures=T, standardized=T, rsquare=T)
```

```
## lavaan (0.5-22) converged normally after  59 iterations
##
##   Number of observations                           301
##
##   Estimator                                         ML
##   Minimum Function Test Statistic              191.509
##   Degrees of freedom                            30
##   P-value (Chi-square)                           0.000
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              918.852
##   Degrees of freedom                            36
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.817
##   Tucker-Lewis Index (TLI)                       0.780
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3790.847
##   Loglikelihood unrestricted model (H1)      -3695.092
##
##   Number of free parameters                         24
##   Akaike (AIC)                                7629.693
##   Bayesian (BIC)                              7718.664
##   Sample-size adjusted Bayesian (BIC)         7642.550
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.134
##   90 Percent Confidence Interval          0.116  0.152
```

```
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.112
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual =~
##     x1                1.000                               0.662    0.588
##     x2                1.227    0.017   71.181    0.000    0.813    0.641
##     x3                0.461    0.013   35.993    0.000    0.305    0.286
##   textual =~
##     x4                1.000                               0.894    0.795
##     x5                1.405    0.020   70.512    0.000    1.256    0.926
##     x6                0.730    0.016   46.157    0.000    0.652    0.672
##   speed =~
##     x7                1.000                               0.550    0.515
##     x8                1.319    0.019   68.858    0.000    0.725    0.715
##     x9                1.282    0.019   67.698    0.000    0.705    0.692
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   visual ~~
##     textual           0.251    0.050    5.001    0.000    0.423    0.423
##     speed             0.170    0.035    4.889    0.000    0.467    0.467
##   textual ~~
##     speed             0.138    0.037    3.756    0.000    0.281    0.281
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.000                               0.000    0.000
##    .x2                0.000                               0.000    0.000
##    .x3                0.000                               0.000    0.000
##    .x4                0.000                               0.000    0.000
##    .x5                0.000                               0.000    0.000
##    .x6                0.000                               0.000    0.000
##    .x7                0.000                               0.000    0.000
##    .x8                0.000                               0.000    0.000
##    .x9                0.000                               0.000    0.000
##     visual            4.945    0.065   76.241    0.000    7.466    7.466
##     textual           3.075    0.064   47.778    0.000    3.439    3.439
##     speed             4.191    0.061   68.343    0.000    7.621    7.621
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .x1                0.830    0.087    9.496    0.000    0.830    0.654
##    .x2                0.949    0.113    8.422    0.000    0.949    0.590
##    .x3                1.044    0.088   11.845    0.000    1.044    0.918
##    .x4                0.465    0.050    9.364    0.000    0.465    0.368
##    .x5                0.263    0.063    4.144    0.000    0.263    0.143
##    .x6                0.516    0.047   11.065    0.000    0.516    0.548
##    .x7                0.837    0.076   10.967    0.000    0.837    0.735
```

```
##    .x8                0.503    0.060    8.328    0.000    0.503    0.489
##    .x9                0.539    0.061    8.818    0.000    0.539    0.521
##     visual            0.439    0.068    6.427    0.000    1.000    1.000
##     textual           0.800    0.076   10.523    0.000    1.000    1.000
##     speed             0.302    0.037    8.192    0.000    1.000    1.000
##
## R-Square:
##                    Estimate
##     x1                0.346
##     x2                0.410
##     x3                0.082
##     x4                0.632
##     x5                0.857
##     x6                0.452
##     x7                0.265
##     x8                0.511
##     x9                0.479

anova(HS7.fit, HS8.fit)

## Chi Square Difference Test
##
##         Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## HS7.fit 24 7535.5 7646.7  85.305
## HS8.fit 30 7629.7 7718.7 191.509      106.2       6  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# compare loadings of HS7.fit and HS8.fit
HS7.pE <- parameterEstimates(HS7.fit)
HS7.pE[HS7.pE$op=="=~",]

##       lhs op rhs   est    se      z pvalue ci.lower ci.upper
## 1  visual =~  x1 1.000 0.000     NA     NA    1.000    1.000
## 2  visual =~  x2 0.554 0.100  5.554      0    0.358    0.749
## 3  visual =~  x3 0.729 0.109  6.685      0    0.516    0.943
## 4 textual =~  x4 1.000 0.000     NA     NA    1.000    1.000
## 5 textual =~  x5 1.113 0.065 17.014      0    0.985    1.241
## 6 textual =~  x6 0.926 0.055 16.703      0    0.817    1.035
## 7   speed =~  x7 1.000 0.000     NA     NA    1.000    1.000
## 8   speed =~  x8 1.180 0.165  7.152      0    0.857    1.503
## 9   speed =~  x9 1.082 0.151  7.155      0    0.785    1.378

HS8.pE <- parameterEstimates(HS8.fit)
HS8.pE[HS8.pE$op=="=~",]

##       lhs op rhs   est    se      z pvalue ci.lower ci.upper
## 1  visual =~  x1 1.000 0.000     NA     NA    1.000    1.000
## 2  visual =~  x2 1.227 0.017 71.181      0    1.194    1.261
## 3  visual =~  x3 0.461 0.013 35.993      0    0.436    0.486
## 4 textual =~  x4 1.000 0.000     NA     NA    1.000    1.000
## 5 textual =~  x5 1.405 0.020 70.512      0    1.366    1.444
## 6 textual =~  x6 0.730 0.016 46.157      0    0.699    0.760
## 7   speed =~  x7 1.000 0.000     NA     NA    1.000    1.000
## 8   speed =~  x8 1.319 0.019 68.858      0    1.281    1.356
## 9   speed =~  x9 1.282 0.019 67.698      0    1.245    1.319

# examine modification indices of HS8.fit
modindices(HS8.fit, minimum.value=10)
```

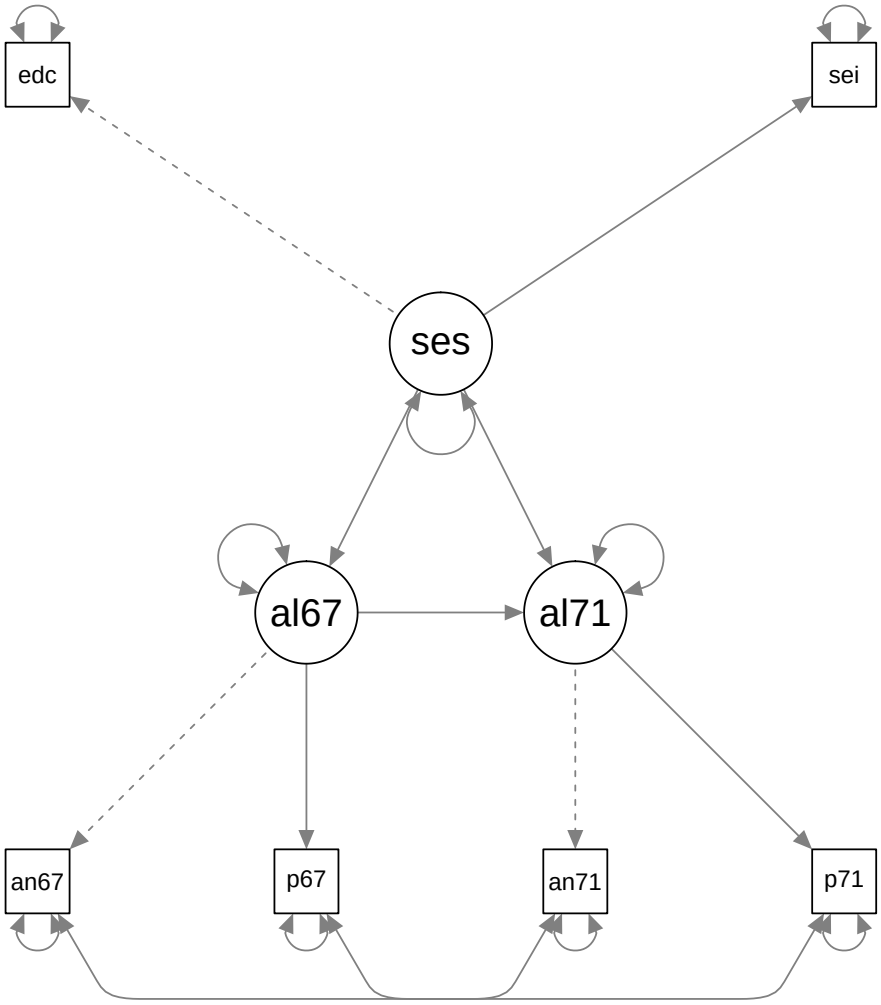
```
##        lhs op rhs     mi    epc sepc.lv sepc.all sepc.nox
## 11      x2 ~1     42.399  7.121   7.121    5.612    5.612
## 12      x3 ~1     27.840 -3.186  -3.186   -2.987   -2.987
## 14      x5 ~1     45.310  1.891   1.891    1.394    1.394
## 15      x6 ~1     34.357 -1.021  -1.021   -1.053   -1.053
## 38  visual =~  x5 23.272  0.345   0.228    0.168    0.168
## 39  visual =~  x6 19.659 -0.184  -0.122   -0.125   -0.125
## 40  visual =~  x7 22.044 -0.614  -0.407   -0.381   -0.381
## 42  visual =~  x9 36.800  0.892   0.591    0.580    0.580
## 43 textual =~  x1 10.399  0.315   0.282    0.250    0.250
## 53   speed =~  x5 32.779  0.418   0.230    0.170    0.170
## 54   speed =~  x6 24.451 -0.218  -0.120   -0.124   -0.124
## 55      x1 ~~  x2 25.557 -0.718  -0.718   -0.502   -0.502
## 56      x1 ~~  x3 20.483  0.285   0.285    0.237    0.237
## 57      x1 ~~  x4 10.080  0.141   0.141    0.111    0.111
## 62      x1 ~~  x9 13.135  0.188   0.188    0.164    0.164
## 71      x3 ~~  x5 12.607 -0.178  -0.178   -0.123   -0.123
## 75      x3 ~~  x9 10.431  0.166   0.166    0.153    0.153
## 76      x4 ~~  x5 20.640 -0.376  -0.376   -0.247   -0.247
## 77      x4 ~~  x6 25.715  0.196   0.196    0.180    0.180
## 88      x7 ~~  x8 17.615  0.224   0.224    0.207    0.207
```

```
# ------------------------------------------------------------------
### test a SEM on a covariance matrix


lower <- '
11.834
6.947 9.364
6.819 5.091 12.532
4.783 5.028 7.495 9.986
-3.839 -3.889 -3.841 -3.625 9.610
-21.899 -18.831 -21.748 -18.775 35.522 450.288'

wheaton.cov <- getCov(lower, names = c("anomia67", "powerless67",
                                       "anomia71", "powerless71",
                                       "education", "sei"))

wheaton.model <- 'ses =~ education + sei
                  alien67 =~ anomia67 + powerless67
                  alien71 =~ anomia71 + powerless71

                  alien71 ~ alien67 + ses
                  alien67 ~ ses

                  anomia67 ~~ anomia71
                  powerless67 ~~ powerless71
                '

wheaton.fit <- sem(wheaton.model, sample.cov = wheaton.cov, sample.nobs = 932)

summary(wheaton.fit, fit.measures=T, standardized = TRUE, rsquare=T)

## lavaan (0.5-22) converged normally after  73 iterations
##
##   Number of observations                           932
##
##   Estimator                                         ML
##   Minimum Function Test Statistic                4.735
```

```
##   Degrees of freedom                                4
##   P-value (Chi-square)                          0.316
##
## Model test baseline model:
##
##   Minimum Function Test Statistic            2133.722
##   Degrees of freedom                               15
##   P-value                                       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                   1.000
##   Tucker-Lewis Index (TLI)                      0.999
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)            -15213.274
##   Loglikelihood unrestricted model (H1)    -15210.906
##
##   Number of free parameters                        17
##   Akaike (AIC)                              30460.548
##   Bayesian (BIC)                            30542.783
##   Sample-size adjusted Bayesian (BIC)       30488.792
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                         0.014
##   90 Percent Confidence Interval          0.000  0.053
##   P-value RMSEA <= 0.05                         0.930
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                          0.007
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   ses =~
##     education         1.000                               2.607    0.842
##     sei               5.219    0.422   12.364    0.000   13.609    0.642
##   alien67 =~
##     anomia67          1.000                               2.663    0.774
##     powerless67       0.979    0.062   15.895    0.000    2.606    0.852
##   alien71 =~
##     anomia71          1.000                               2.850    0.805
##     powerless71       0.922    0.059   15.498    0.000    2.628    0.832
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   alien71 ~
##     alien67           0.607    0.051   11.898    0.000    0.567    0.567
##     ses              -0.227    0.052   -4.334    0.000   -0.207   -0.207
##   alien67 ~
##     ses              -0.575    0.056  -10.195    0.000   -0.563   -0.563
```

```
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .anomia67 ~~
##    .anomia71          1.623    0.314    5.176    0.000    1.623    0.356
##  .powerless67 ~~
##    .powerless71       0.339    0.261    1.298    0.194    0.339    0.121
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .education          2.801    0.507    5.525    0.000    2.801    0.292
##    .sei              264.597   18.126   14.597    0.000  264.597    0.588
##    .anomia67           4.731    0.453   10.441    0.000    4.731    0.400
##    .powerless67        2.563    0.403    6.359    0.000    2.563    0.274
##    .anomia71           4.399    0.515    8.542    0.000    4.399    0.351
##    .powerless71        3.070    0.434    7.070    0.000    3.070    0.308
##     ses                6.798    0.649   10.475    0.000    1.000    1.000
##    .alien67            4.841    0.467   10.359    0.000    0.683    0.683
##    .alien71            4.083    0.404   10.104    0.000    0.503    0.503
##
## R-Square:
##                    Estimate
##     education         0.708
##     sei               0.412
##     anomia67          0.600
##     powerless67       0.726
##     anomia71          0.649
##     powerless71       0.692
##     alien67           0.317
##     alien71           0.497
```

```
library(semPlot)
semPaths(wheaton.fit, title=F, curvePivot=T)
```


edc
sei
ses
al67
al71
an67
p67
an71
p71